

fectly easy in every respect : which rule is so comprehensive, that it also includes the questions of the compound rule, or rule-of-five ; and that not only those which are commonly given in books, wherein the statings are either both direct, or one direct and the other inverse, but also those wherein the statings are both indirect.

The fractions are pretty largely treated of ; and particularly the abbreviating part ; since the generality of boys are seldom taught them for any other purpose but that of abbreviating the operations in all the other rules. The advantage of fractions is so great, that I dare affirm it, a person who is well acquainted with them, will perform as many calculations as four or five who are not. In decimals the separating points are placed against the upper part of the figures ; which prevents them from being mistaken for stops or pauses in the reading ; the hint of which I had from some tables in Sir *Isaac Newton's* optics. And from Mr. *Colson* I have inserted a very expeditious method for converting vulgar fractions into decimals consisting of many figures.

In compound interest, where the time, at which the interest is supposed to be payable, is some part of a year, I have, in my calculations, accounted the rate corresponding to that time, the same part of the rate for the whole year : thus, at 5 *per cent. per annum*, the rate for half-yearly payments I make $2\frac{1}{2}$, and for quarterly payments, $1\frac{1}{4}$, &c. I say this here to shew upon what supposition these examples are calculated, as it is contested whether this method ought to be used or not.

In

P R E F A C E.

• In the extraction of roots I have given a new, general, and very expeditious method, by which the third and higher roots may be found without any of that intolerable labour attending the common methods.

The proportions are very largely handled, because they are so slightly treated of in common books, and the rules are expressed both algebraically and in words at length. I have also inserted the rules of false, because of their great use in resolving many problems, not only in arithmetic, but likewise in algebra, geometry, and in short, all the branches of mixt mathematics. For where there is any proportion of the things sought and those assumed to represent them, with their similar results, there the single rule gives a perfect and the most natural solution: thus, if it be required to find the side of an equilateral triangle whose area is 100 square feet; as all equilateral triangles are similar, and similar triangles are as the squares of their like, or corresponding sides, if we assume an equilateral triangle whose side is of any given length, as suppose of 1 foot, and by any known method calculate its area, which will be $\frac{\sqrt{3}}{4}$; we may

hence find the thing required: thus $\frac{\sqrt{3}}{4} : 1^2 :: 100 :$

400
 $\sqrt{3}$ the square of the side sought; and therefore, by

taking the square root; we have $20 \times 3^{\frac{1}{4}} =$ the required side of the triangle. And in the same manner may the reverse of this problem be performed, viz. having given the side of a regular figure, to find its area: for,

for, by making the same supposition as before, and inverting the proportion, we shall then have the area for the fourth term. Now in this very manner do the writers on mensuration teach us to find the areas of regular figures; and for this purpose they take the trouble to form tables of the areas of several figures whose sides are each equal to 1. And are not all the cases of trigonometry wrought by the same rule? for the writers of it give us large tables containing the parts of all sorts of right-angled triangles, calculated to a constant radius (1); so that no triangle can be proposed, but one similar to it may be there found in known terms; and therefore by forming the homologous parts into a proportion, we arrive at the knowledge of the required parts of the triangle proposed. —And where we cannot find such proportion as the single rule requires, which happens in many intricate problems in most parts of the mathematics, yet there the double rule will approximate to the things required by a very easy and quick process: and particularly in exponential equations, scarce any method beside this of *Trial and Error* is used to find the value of the unknown quantities.

But I have omitted the rule called permutation, as it is of no use in common arithmetic, and of but very little in any other part of the mathematics. I have omitted also duodecimal arithmetic, as its place is better supplied, in all cases, by the common decimal scale of numbers.

To this edition are added the roman notation, a promiscuous collection of questions, and an appendix of retail book-keeping.

C O N T E N T S.

	Page.
N OTATION	1
A Synopsis of the Roman Notation	3
Simple Addition	4
Subtraction	5
Multiplication	6
Division	9
Reduction	13
Compound Addition	19
Subtraction	25
Multiplication	27
Division	32
Rule-of-Three	34
Rule-of-Five	38
Vulgar Fractions	40
Reduction of Vulgar Fractions	41
Addition of Vulgar Fractions	49
Subtraction of Vulgar Fractions	50
Multiplication and Division of Vulgar Fractions	51
Rule-of-Three in Vulgar Fractions	52
Rule-of-Five in Vulgar Fractions	54
Decimal Fractions	Ib.
Addition and Subtraction of Decimals	55
Multiplication of Decimals	56
Division of Decimals	57
Reduction of Decimals	60
Rule-of-Three in Decimals	63
Rule-of-Five in Decimals	64
Practice	Ib.
Tare and Tret	72
Bills of Parcels, Book-Debts, &c.	75
Simple Interest	80
Compound Interest	85
Rebate or Discount	86
Equation of Payments	88
Single Fellowship	89
	Double

C O N T E N T S.

Double Fellowship	—	—	—	91
Barter	—	—	—	93
Loss and Gain	—	—	—	95
Exchange	—	—	—	97
Alligation	—	—	—	101
Involution	—	—	—	106
Evolution	—	—	—	109
Of Proportion in general	—	—	—	118
Arithmetical Progression	—	—	—	120
Geometrical Progression	—	—	—	130
Single Position	—	—	—	139
Double Position	—	—	—	141
A Promiscuous Collection of Questions	—	—	—	144
Appendix of Book-keeping	—	—	—	152

A
COMPLETE SYSTEM
OF
Practical Arithmetic.

PRACTICAL ARITHMETIC is the art of numbering, or of performing calculations by numbers.

NOTATION.

NOTATION is the expressing of any proposed number, either by words, or characters.

All numbers are expressible by these ten characters or figures, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, or cypher: and the usual method of notation by figures is so contrived, that any character is increased in its value in a tenfold proportion for every place it is removed to the left among the other figures with which it is connected; so in these figures 333, the first 3 (reckoning from the right to the left) is 3 ones, but the second is 3 tens, and the third is 3 hundreds; also in these 2759, the 9 represents 9 ones, but the 5 represents 5 tens, the 7 is 7 hundreds, and the 2 is 2 thousand. And though the cypher signify nought of itself, yet when put on the right of any of the other figures, it increaseth their value in the same tenfold proportion above described; thus, though 2 standing alone, or in the first place, represent only 2 ones, yet when a cypher is wrote on the right of it, thus 20, it represents 2 tens or twenty; and if another cypher be affixed, thus 200, it will represent 2 hundreds.

For the more easy reading of large numbers, when they are expressed by figures, they are divided, from the right hand towards the left, into periods and half-periods, each half-period consisting of three figures; the common name of the first period being units or ones; of the second, millions; of the third, billions; of the fourth, trillions,

&c. Also the first half of any period is so many ones of it, but the latter half is so many thousands of it.

The following example exhibits a summary of this whole doctrine.

Quintillions		Quadrillions		Trillions		Billions		Millions		Units					
th.	un.	th.	un.	th.	un.	th.	un.	th.	un.	c.	x	t.	c.	x.	u.
473,	829.	759,	761.	235,	871.	296,	473.	913,	651.	4	3	7	2	5	6.

Note, The first nine characters are called digits, and sometimes significant figures, to distinguish them from the cypher, which of itself is insignificant. Also, a number expressing a quantity of one name or denomination, is called a simple number, as 20 pounds, or 17 gallons, or 5 days; and that representing a quantity of several names, is called a compound number, as 13 pounds 5 shillings and 6 pence, or 17 gallons and two pints, or 3 hours and 50 minutes.

I. *Having any number proposed in words, to express the same in figures.*

R U L E.

Write down cyphers to so many periods and places as are named in the given number; then beginning at the left, observe at each place what significant figure is named, and, taking away the cypher, write the significant figure in its place,

E X A M P L E S.

1. Express in figures, Four thousand, one hundred, and seventy-three.
2. Write down in figures, Twenty-three millions, two hundred and six thousand, nine hundred and thirty.
3. Write in figures, Four thousand and twenty-five millions, one hundred and three thousand, and six.
4. Express in figures, Two hundred seventeen thousand and fifty millions, eight thousand, seven hundred, and sixteen.
5. Write down in figures, Seventy thousand billions, one hundred three thousand and fifty millions, three thousand and eight.
6. Express in figures, Eight hundred trillions, one hundred seventy-five thousand seven hundred and forty-eight billions, three hundred thousand millions, five thousand, and seventy.

II. *Having any number expressed in figures, to read the same, or express it in words.*

R U L E.

R U L E.

Divide the figures in the given number, as in the general example, into periods and half-periods, by any convenient marks; then, beginning at the left, the figures are thus read; viz. the first figure of each half-period is named by itself with the word *hundreds*, but the other two are named together; and at the end of the first half of each period, the word *thousands* is named, but at the end of the other half, the common name of the whole period, except it be the units period, whose name is not expressed

E X A M P L E S.

1. Let it be required to express in words, 17359.
2. Write down in words, 7301462.
3. Write in words, 3920500706.
4. Express in words, 102003000400.
5. Write in words, 2073000091630702.
6. Write down in words, 503002786940003.

A SYNOPSIS of the ROMAN NOTATION.

- 1 = I
 2 = II As often as any character is repeated, so many times its value is repeated.
 3 = III
 4 = IIII or IV A less character before a greater diminishes its value.
 5 = V
 6 = VI A less character after a greater increases its value.
 7 = VII
 8 = VIII
 9 = IX
 10 = X
 50 = L
 100 = C
 500 = D or IↃ: for every Ↄ affixed, this becomes 10 times so many.
 1000 = M or CIↃ: for every C & Ↄ, put one at each end, it becomes 10 times so much.
 2000 = MM
 5000 = IↃↃ or V̄ A line over any number, increases it 1000 fold.
 6000 = VĪ
 10000 = X̄ or CCIↃↃ

$$50000 = \text{I} \overline{\text{C}} \overline{\text{C}} \overline{\text{C}}$$

$$60000 = \overline{\text{LX}}$$

$$100000 = \overline{\text{C}} \text{ or } \text{CCC} \overline{\text{I}} \overline{\text{C}} \overline{\text{C}}$$

$$1000000 = \overline{\text{M}} \text{ or } \text{CCCC} \overline{\text{I}} \overline{\text{C}} \overline{\text{C}} \overline{\text{C}}$$

$$2000000 = \overline{\text{MM}}$$

&c.

SIMPLE ADDITION.

SIMPLE Addition is the finding of one simple number equal to several simple numbers taken all together.

The number which is equal to several taken together, is called their *sum*.

Simple Addition may be performed by this

R U L E.

1. Place the several numbers, to be added, underneath each other, so that the figures of the same name, with respect to units, tens, &c. may be strait under each other.

2. Draw a line under the lowest number, then add up the column of units, and consider how many tens are in the sum, for which you must carry so many ones to the next column, writing down only the excess over and above the tens below the line streight under its proper column.

3. Add all the columns in the same manner, and the figures below the line will express the sum required.

There are several methods used to prove addition, but I think that working the question over again is as good as any.

E X A M P L E S.

1. What is the sum of 37, 509, 7126, 17630, and 459273?

2. Required the sum of 3579, 41, 96120, 725, 11, 1820, 5, and 720139?

3. What is the sum of 2591, 720396, 14, 259, 6, 370214, 9740, 53, 1692, and 137?

4. How many days are in the twelve calendar months?

5. Sup-

5. Suppose that from London to Hatfield is 20 miles, from thence to Stilton 57 miles, thence to Newark 48 miles, thence to Doncaster 37 miles, thence to Northalerton 62 miles, thence to Durham 34 miles, and from thence to Newcastle 15 miles; how many miles are between London and Newcastle.

6. A person dying, left to his widow 1500 pounds, to his eldest son he left 30500, and to each of his other two sons 3456; also 2700 to each of his three daughters, besides 751 pounds in other small legacies; what did he die possessed of?

SIMPLE SUBTRACTION.

SIMPLE Subtraction is the finding how much one simple number exceeds another, or the taking a less simple number out of a greater.

The number to be subtracted is called the *subtrahend*, and that out of which it is to be taken is called the *minuend*; also the number remaining after the one is taken out of the other, is named their *difference*.

Simple Subtraction is performed by the following

R U L E.

1. Place the subtrahend under the minuend according to the directions given in addition, and draw a line below them.

2. Begin at the right, and subtract each under figure from that which stands above it, writing the remainder streight under them below the line; so shall all the remainders together express the difference required.

3. But when any under figure exceeds that which is above it, conceive 10 to be added to the upper, and subtract the under from the sum; but in this case, you must add 1 to the next under figure, before you subtract it.

To prove Subtraction,

Add the difference and subtrahend together, and the sum will be equal to the minuend when the operation is right.

EXAMPLES.

1. What is the difference between 1735 and 1897348?
2. How much does 510312 exceed 7953?
3. How much is 30491 less than 57321469?
4. Suppose that from London to Edinburgh (by way of Newcastle) is 393 miles, and that from London to Newcastle is 273 miles; how many miles are between Newcastle and Edinburgh?
5. How much is A older than B, A being born in the year 1701, and B in 1739?
6. How much is C, whose age is 71, older than D, whose age is 34?

SIMPLE MULTIPLICATION.

SIMPLE Multiplication is the finding of a simple number which shall contain any given simple number a certain proposed number of times; and is therefore a compendious method of addition.

The two proposed numbers are, in general, termed the *factors* of the multiplication; but in particular, that which is to be multiplied, is called the *multiplicand*, and that you multiply by, the *multiplier*; also the number to be found from the operation is named the *product* of the two factors.

Simple Multiplication may be performed by the two following

RULES.

I. *To multiply by a single figure, or by any number in the first line of the following table of products.*

Begin at the right of the multiplicand, and multiply each figure in it by the multiplier, writing down the whole of such products as are less than ten; but for such as are just equal to a certain number of tens, write down 0, and carry one for each 10 to the next product; and for such as exceed a certain number of tens, write down the excess, and carry for the tens as before.

II. *To multiply by a number consisting of several figures.*

1. Write it below the multiplicand, and find the product for each figure in it as in the first case, not regarding

ing in what order they are found, provided the first figure of each stand straight below its respective multiplier.

2. Add all the lines of products together in the same order as they stand, and the sum will be the whole product required.

Multiplication may be proved by casting out the nines, but the most natural and certain way is by division.

TABLE of PRODUCTS.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3		9	12	15	18	21	24	27	30	33	36
4			16	20	24	28	32	36	40	44	48
5				25	30	35	40	45	50	55	60
6					36	42	48	54	60	66	72
7						49	56	63	70	77	84
8							64	72	80	88	96
9								81	90	99	108
10									100	110	120
11										121	132
12											144

EXAMPLES.

1. Multiply 7130492685 by 4.
2. What is the product of 203516748 and 7?
3. What is the product of 15731800296 and 9?
4. What is the product of 58013002716 and 11?
5. What is the product of 932701568 and 12?
6. Required the product of 273580961 and 23.
Answer 6292362103.
7. Required the product of 6241578309 and 37.
Ans. 230938397433.
8. What is the product of 5318625074 and 43?
Ans. 228700878182.
9. What is the product of 751900368 and 51?
Ans. 38346918768.
10. What is the product of 402097316 and 195?
Ans. 78408976620.
11. What

8 SIMPLE MULTIPLICATION.

11. What is the product of 82164973 and 3027?
Ans. 248713373271.
12. What is the product of 16358724 and 704006?
Ans. 11516639848344.
13. What is the prod. of 921760035 and 520007091?
Ans. 479321754400408185.
14. What is the prod. of 38015732 and 400700065?
Ans. 15232906283422580.

CONTRACTIONS.

I. When there are cyphers at the right of one or both factors, proceed as before, neglecting the cyphers, and to the right of the product, place as many cyphers as are in both factors.

EXAMPLES.

1. Multiply 718603400 by 57. Ans. 40960393800.
2. Required the product of 70100 and 9001635.
Ans. 631014613500.
3. What is the product of 9030100 and 21000?
Ans. 189632100000.
4. What is the product of 7030 and 815036000?
Ans. 5729703080000.

II. So that, if any number is to be multiplied by 1 with cyphers annexed, the product will be equal to the multiplicand with so many cyphers as are in the multiplier.

EXAMPLES.

1. The product of 71 and 10 is 710.
2. The product of 2103 and 100 is
3. The product of 503700 and 1000 is

III. When the multiplier is the product of two or more numbers in the table, it is often of advantage to multiply continually by those numbers instead of it.

EXAMPLES.

1. Required the product of 51307298 and 56, or 7 times 8.
Ans. 2873208688.
2. What is the product of 31704592 and 36?
Ans. 1141365312.
3. What is the product of 29753804 and 72?
Ans. 2142273888.
4. What

4. What is the product of 7128368 and 96?

Anf. 684323328.

5. What is the product of 61835720 and 1320?

Anf. 81623150400.

IV. When some of the figures of the multiplier may be produced by multiplying some others of them by any number, it is much easier and more concise, after having obtained the product of the less, to multiply that product by the same number for the product of the greater, than to proceed by the common method.

Note, This holds as well when the less numbers are on the left as when they are on the right of the greater; for, by the general rule, the products of the figures of the multiplier may be taken in any order.

1. Multiply 35802916 by 93. Anf. 3329671188.

2. Required the product of 910738060 and 48.

Anf. 43715426880.

3. What is the product of 61370913 and 96488?

Anf. 5921556653544.

4. What is the product of 13861470 and 52575?

Anf. 728766785250.

5. What is the product of 71380164 and 27354?

Anf. 1952533006056

SIMPLE DIVISION.

SIMPLE Division is the finding how oft one simple number is contained in another; or the dividing of any given simple number into any proposed number of equal parts.

The containing number, or number to be divided, is called *dividend*.

The contained number, or the number of parts into which the dividend is divided, is called *divisor*.

The number of times the dividend contains the divisor, or the number which expresses one of the equal parts, is called *quote* or *quotient*: thus

Dividend

Divisor 3) 12 (4 Quote

N. B. Division is a compendious subtraction, the quote being the number of subtractions in the operation.

Simple

Simple Division may be performed by the following
R U L E.

1. Having wrote down the divisor and dividend in the form above, consider if the divisor be less than, or equal to, the same number of the left-hand figures of the dividend; if so, write the figure expressing the number of times it is contained in the quote; but if not, take one place more of the dividend figures than are in the divisor, and write the number of times they contain it in the quote as before.

2. Multiply the divisor by the quotient figure.

3. Subtract the product from the said dividend figures.

4. To the remainder affix the next dividend figure, and write in the quote the number of times the divisor is contained in this number; multiply the divisor into the last quotient figure, and subtract the product from the last mentioned number; then proceed as before from the beginning of this article, till all the dividend figures be used.

Note 1. It is sometimes troublesome to find how often the quote is contained in the several dividends; but part of the trouble will be saved by observing, that when any product exceeds its dividend, the quotient figure belonging to such product must be lessened till the product be equal to, or less than its dividend; again, if, after subtracting the product from its dividend, the remainder be equal to, or exceed the divisor; the quotient figure must be increased till the remainder be less than it.

2. To complete the quote, put the last remainder (if any) at the end of it, above a small line with the divisor below it.

To prove division,

Multiply the quote into the divisor, to the product add the remainder, and the sum will be equal to the dividend when the work is right.

EXAMPLES.

1. Divide 73146085 by 4. — Ans. 18286521 $\frac{1}{4}$.
2. What is the quote of 5317986027 divided by 7?
Ans. 759712289 $\frac{3}{7}$.
3. What is the quote of 570196382 by 12?
Ans. 47516365 $\frac{2}{3}$.
4. How often is 37 contained in 74638105?
Ans. 2017246 $\frac{1}{7}$ times.
5. How often does 137896254 contain 97?
Ans. 1421610 $\frac{3}{4}$ times.
6. Di-

6. Divide 35821649 into 764 equal parts.

Anf. They are each equal $46886\frac{4}{7}$.

7. What is the quote of 72091365 by 5201?

Anf. $13861\frac{3}{4}$.

CONTRACTIONS.

I. Division by a single figure, or by any figure in the first line of the table of products, may be expeditiously perform'd by multiplying and subtracting mentally, and writing down only the quote below the dividend.

EXAMPLES.

$$3) 56103961$$

$$4) 52019675$$

Quote 187013204

$$5) 1370192$$

$$6) 38172940$$

$$7) 81390627$$

$$8) 23718620$$

$$9) 4308196$$

$$10) 7803196$$

$$11) 5701423$$

$$12) 2798013$$

II. When the divisor has cyphers on the right of it, you may strike them off and divide without them; but the same number of figures must be struck off from the right of the dividend and affixed to the last remainder.

EXAMPLES.

$$2,0) 370419,6$$

$$12,00) 718306,15$$

Divide 3108690170 by 7100. — Ans. $437843\frac{47}{100}$.

What is the quote of 7380964 by 23000?

Ans. $320\frac{2264}{11500}$.

What is the quote of 2304109 by 5800?

Ans. $397\frac{1109}{5800}$.

III. Whence to divide by 1 with any number of cyphers annexed, you need only strike off from the right of the dividend so many figures as the divisor contains cyphers; which figures so struck off will be the remainder, and those on the left, the quote.

EXAMPLES.

5138602 divided by 100 is equal $51386\frac{2}{100}$.

2701483 by 1000 is

3702140 by 100 is

IV. When the divisor is the product of two or more small numbers, it is much easier to divide continually by those numbers than by the whole divisor at once.

Note, If there be any remainders after such divisions, multiply the last remainder by the preceding divisor, and to the product add the remainder belonging to the same divisor; then multiply the sum by the next preceding divisor, and to the product add its corresponding remainder: proceed in the same manner through all the divisors and remainders, so shall the last sum be the remainder as if the division had been performed at once.

After the operation described in this Note is begun, it must be continued according to the description, tho' some of the preceding divisions have no remainders.

EXAMPLES.

1. Divide 31046835 by 56, or 7 times 8.

Quote $554407\frac{3}{56}$.

2. Divide 7014596 by 72. — Quote $97424\frac{4}{9}$.

3. Divide 5130652 by 132. — Quote $38868\frac{76}{132}$.

4. Divide 83016572 by 240. — Quote $345902\frac{23}{240}$.

V. When you are pretty ready in division, you may, even in the largest divisions, subtract each figure of the product as you produce it, and write down only the remainders.

REDUCTION.

13

EXAMPLES.

1. Divide 3104679 by 833.
833) 3104679 (3727 $\frac{44}{833}$.

6056

2257

5919

88

2. Divide 79165238 by 238. — Quote 332627 $\frac{17}{238}$.
3. Divide 29137062 by 5317. — Quote 5479 $\frac{112}{5317}$.
4. Divide 62015735 by 7803. — Quote 7947 $\frac{184}{7803}$.

REDUCTION.

REDUCTION is the conversion of numbers from one name to another, but still retaining the same value.

If the reduction be to a less name, it is commonly called reduction *descending*; but if to a greater, reduction *ascending*.

RULE.

Consider how many of the less name concerned make one of the greater, and by that number multiply the given number if the reduction be descending, but divide if ascending, and the product or quote will be of the other name.

Note 1. When there are names between the proposed and required, it is best to reduce the proposed to the next less or greater name, and this to the next less or greater again, and so on, till you have reduced it to the name required.

2. When, in reduction descending, the proposed is a compound number, you must add, or take in the small numbers in the names below the greatest, to the same names as you proceed in the reduction.

3. When, in reduction ascending, you have any remainders after dividing, they will have the same names as their respective dividends, and may be placed after the last quote, according to the order of their names, the greater first; so shall the compound number thus formed be the answer.

REDUCTION. Of MONEY.

Farthings	Pence	Shilling	pounds
4	1	1	1
48	12	1	1
960	240	20	1

Note 1. This and the following tables are to be understood thus: the words at the top are the names

of all the numbers streight below them; and all the numbers upon the same line, from left to right, are of equal value: thus in the last line of this table, 960 farth. 240 pence, 20 shill. and 1 pound are all equal to each other.

2. *l* denotes pounds, *s* denotes shillings, and *d* denotes pence.
3. $\frac{1}{4}$ denotes 1 farthing, or 1 quarter of any thing.
- $\frac{1}{2}$ ——— a half-penny, or a half of any thing.
- $\frac{3}{4}$ ——— 3 farthings, or 3 quarters of any thing.

EXAMPLES.

1. How many shillings and pence are in 23 *l*.?
Ans. 460 *s.* 5520 *d.*
2. Reduce 5520 pence to shillings and pounds.
Ans. 460 *s.* 23 *l.*
3. Reduce 351 *l.* 13 *s.* 04 *d.* to farthings.
Ans. 337587 farthings.
4. How many pounds, &c. are in 337587 farthings?
Ans. 351 *l.* 13 *s.* 04 *d.*
5. In 35 guineas how many farthings?
Ans. 35280 farthings.
6. In 35280 farthings how many guineas?
Ans. 35 guineas.
7. How many crowns, shillings, groats and pence are in 50 pounds? *Ans.* 200 *cr.* 1000 *s.* 3000 *gr.* 12000 *d.*
8. Reduce 12000 pence to groats, shillings, crowns, and pounds. *Ans.* 3000 *gr.* 1000 *s.* 200 *cr.* 50 *l.*

Of TROY WEIGHT.

Grains	Pennyweights	Ounces	Pound
24	1	1	1
480	20	1	1
5760	240	12	1

Note, By this weight are weighed jewels, gold, silver, corn bread, and liquors.

EXAMPLES.

1. How many ounces, pennyweights, and grains are in 37 *lb.* ——— *Ans.* 444 *oz.* 8880 *dwt.* 213120 *grs.*
2. Re-

2. Reduce 213120 grains to *lbs.* — *Ans.* 37 *lb.*
 3. In 59 *lb.* 13 *dwt.* 5 *grs.* how many grains? *Ans.* 340157 *grs.*
 4. In 340157 grains, how many *lbs.* &c. *Ans.* 59 *lb.* 13 *dwt.* 5 *grs.*

Of APOTHECARIES WEIGHT.

Grains	Scruples			
20	1	Drams		
60	3	1	Ounces	
480	24	8	1	Pound
5760	288	96	12	1

Note, This weight is so called, because the apothecaries use it in compounding

their medicines; but they buy and sell their drugs by avoirdupois weight. Apothecaries is the same as troy wt. having only some different divisions.

EXAMPLES.

1. In 17 *lb* how many ounces, drams, and scruples? *Ans.* 204 *oz.* 1632 *dr.* 4896 *scr.*
 2. How many *lbs.* are in 4896 scruples? — *Ans.* 17 *lb.*
 3. In 231 *lb.* 3 *oz.* and 5 *grs.* how many grains? *Ans.* 1332005 *grs.*
 In 1332005 grains, how many *lb*? *Ans.* 231 *lb.* 3 *oz.* 5 *grs.*

Of AVOIRDUPOIS WEIGHT.

Drams	Ounces				
16	1	Pounds			
256	16	1	Quarters		
7168	448	28	1	Hundred	
28672	1792	112	4	1	Ton
573440	35840	2240	80	20	1

Note, By this weight are weighed all things of a coarse or drossy nature; such as grocery and chandlers wares, and all metals, except gold and silver.

EXAMPLES.

1. In 15 tons, how many *c. qrs.* and *lb*? *Answer* 300 *c.* 1200 *qrs.* 33000 *lb.*
 2. Reduce 33600 *lb.* to tons. — *Ans.* 15 tons.
 3. In 9 *c.* 5 *lb.* how many ounces? — *Ans.* 16208 *oz.*
 C 2 4. How

4. How many *c.* are in 16208 oz? — *Anf.* 9 *c.* 5 *lb.*
 5. In 35 *ton*, 17 *c.* 1 *qr.* 23 *lb.* 7 *oz.* 13 *dr.* how many
 drams? — — — *Anf.* 20571005 *dr.*
 6. Reduce 20571005 drams to tons.
Anf. 35 *t.* 17 *c.* 1 *qr.* 23 *lb.* 7 *oz.* 13 *dr.*

OF LONG MEASURE.

Inches	Feet				
12	1	Yards			
36	3	1	Poles		
198	16½	5½	1	Furlongs	
7920	660	220	40	1	Mile
63360	5280	1760	320	8	1

Note, An inch is supposed equal to 3 barley corns in length.

4 inch.—a hand.

6 feet.—a fathom

3 miles.—a league.

60 nautical or geographical miles.—a degree, or 69½ statute miles nearly.

Also 360 degrees, or 25000 miles nearly, is the circumference of the earth.

EXAMPLES.

1. How many inches are between London and Newcastle, or in 273 miles? — *Anf.* 17297280 *inch.*
 2. In 17297280 inches, how many miles?
Anf. 273 *miles.*
 3. Reduce 15 *mls.* 1 *furl.* 3 *yds.* into inches.
Anf. 364428 *inch.*
 4. In 364428 inches, how many miles?
Anf. 15 *mls.* 1 *fur.* 3 *yds.*
 5. Reduce 2 *mls.* 1 *furl.* 8 *pls.* 3 *yds.* 2 *inc.* into inches.
Anf. 136334 *inch.*
 6. In 136334 inches, how many miles, &c?
Anf. 2 *mls.* 1 *furl.* 8 *pls.* 3 *yds.* 2 *inch.*

OF CLOTH MEASURE.

Inches	Nails		
2	1	Quartreils	
9	4	1	Yard
36	16	4	1

Note,

REDUCTION.

17

Note, 3 qrs. are equal to an ell flemish.

5 — — english.
6 — — french.
4 qrs. $1\frac{1}{2}$ inch. — scots.

EXAMPLES.

1. In 37 yds. how many qrs. and nails?
Ans. 148 qrs. 592 nails.
2. How many yds. are in 592 nails? — *Ans.* 37 yds.
3. Reduce 15 yds. 3 qrs. 1 nl. to nails. *Ans.* 253 nails.
4. How many yds. are in 253 nails?
Ans. 15 yds. 3 qrs. 1 nl.
5. In 73 ells flemish, how many qrs? — *Ans.* 219 qrs.
6. How many ells flem. are in 219 qrs?
Ans. 73 ells flem.
7. Reduce 17 ells eng. 3 qr. to nails. — *Ans.* 352 nails.
8. In 352 nails, how many ells eng.?
Ans. 17 el. eng. 3 qr.

Of SQUARE, or LAND MEASURE.

Square inch.	Sqr. feet				
144	1	Sqr yds.			
1296	9	1	Sqr. poles		
39204	272 $\frac{1}{4}$	30 $\frac{1}{4}$	1	Roods	
1568160	1890	1210	40	1	Acre
6272640	43660	4840	160	4	1

EXAMPLES.

1. In 15 Acres, how many poles. — *Ans.* 2400 poles.
2. How many acres are in 2400 poles? — *Ans.* 15 acres.
3. Reduce 27 a. 1 r. 32 p. into poles — *Ans.* 4392 poles.
4. Reduce 4392 poles into acres. — *Ans.* 27 a. 1 r. 32 p.

Of WINE MEASURE.

Pints	Gallons					
8	1	Tierces				
336	42	1	mhds.			
504	63	1 $\frac{1}{2}$	1	Punch.		
672	84	2	1 $\frac{1}{3}$	1	Pip. or But	
1008	126	3	2	1 $\frac{1}{2}$	1	Run
2016	252	6	4	3	2	1

Note, 231 solid inch. — a gallon.

30 gall. — an anchor.

18 gall. — a muidlet.

$31\frac{1}{2}$ gall. — a barrel.

By this measure, wines, brandies, spirits, perry, cyder, mead, vinegar, oil, and honey are measured.

EXAMPLES.

1. In 19 *bbds.* of wine, how many pints? *Ans.* 9576 pints.
2. How many *bbds.* are in 9576 pints of wine? *Ans.* 19 *bbds.*
3. Reduce 13 *tuns*, 1 *pipe*, 1 *bbd.* 17 *gall.* 5 *pts.* to pints.
Ans. 27861 pints.
4. Reduce 27861 pints to *tuns*.
Ans. 13 *tuns*, 1 *pipe*, 1 *bbd.* 17 *gall.* 5 *pts.*

Of ALE and BEER MEASURE.

Pints	Gall.				
8	1	Firk.			
68	$8\frac{1}{2}$	1	Kilderk.		
136	17	2	1	Bar.	
272	34	4	2	1	Hhds
408	51	6	3	$1\frac{1}{2}$	1

Note, The ale gallon contains 182 cubic inches.

In London, the ale firkin, contains 3 gal.

and the beer firkin 9; the other measures above it being decreased and increased in the same proportion.

EXAMPLES.

1. In 13 *bbds.* of ale, how many gallons? — *Ans.* 663 *gall.*
2. How many *bbds.* are in 663 gallons of ale? *Ans.* 13 *bbds.*
3. How many pints are in 1 *bar.* 1 *fir.* 3 *pts.* of ale?
Ans. 343 pints.
4. Reduce 343 pints of ale to barrels.
Ans. 1 *bar.* 1 *fir.* 3 *pts.*

Of DRY MEASURE.

Pints	Gall.						
8	1	Pecks					
16	2	1	Bush.				
64	8	4	1	Combs			
256	32	16	4	1	Quar.		
512	64	32	8	2	1	Weyls	
2560	320	160	40	10	5	1	Last
5120	640	320	80	20	10	2	1

Note,

REDUCTION.

19

Note, The gall. dry measure, contains 268 $\frac{1}{2}$ cubic inches. At London, 36 bushels of coals make a chaldron. A bushel water measure is 5 pecks.

By this measure, all dry wares, such as corn, seeds, fruits, roots, sand, salt, coals, oysters, muscles, cockles, &c. are measured.

EXAMPLES.

1. In 128 qrs. how many pecks? — *Ans.* 4096 pecks.
2. How many qrs. are in 4096 pecks? — *Ans.* 128 qrs.
3. In 3 lasts, 5 qrs. 3 bush. how many gallons?
Ans. 2264 gal.
4. Reduce 2264 gallons to lasts. *Ans.* 3 gls. 5qr. 3bush.

OF TIME.

Minutes	Hours			
60	1	Days		
1440	24	1	Weeks	
10080	168	7	1	Month
40320	672	28	4	1

Note, The minute is divided into 60 seconds, and the second may be supposed to be divided into

60 thirds, and these again into 60 fourths, &c.

EXAMPLES.

1. How many minutes are in 1763 months?
Ans. 71084160 min.
2. In 71084160 minutes, how many months?
Ans. 1763.
3. How many seconds are in a solar year, or 365 ds. 5 hrs. 48 min. 58 sec.? — — *Ans.* 31556938 sec.
4. In 31556938 seconds, how many days, &c.? *Ans.* 365 ds. 5 hrs. 48 min. 58 sec.
5. In a lunar month, or 29 ds. 12 hrs. 45 min. how many seconds? — — — *Ans.* 2551500 sec.
6. Reduce 2551500 seconds to days.
Ans. 29 ds. 12 hrs. 45 min.

COMPOUND ADDITION.

COMPOUND Addition is the finding of the sum of several compound numbers.

RULE.

1. Place the numbers of the same denomination under each other according to the directions given in simple addition.
2. Add

2. Add up the figures in the lowest denomination as in simple addition.

3. Find how many ones of the next higher denomination are contained in the sum, by dividing it by so many as of this name make one of the next, or any other way.

4. Write the remainder or overplus underneath, and carry the ones to the figures in the next denomination, whose sum you must find and proceed with as before; and so on, through all the denominations to the highest, whose sum must be all wrote down, which, together with the several remainders, will express the sum required.

Note, Addition of money may be performed by the general rule, or by the help of the following

PENCE TABLES.

<i>d.</i>		<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>
20	—	1	8	2	—
30	—	2	6	3	—
40	—	3	4	4	—
50	—	4	2	5	—
60	—	5	-	6	—
70	—	5	10	7	—
80	—	6	8	8	—
90	—	7	6	9	—
100	—	8	4	10	—
110	—	9	2	11	—
120	—	10	-	12	—

EXAMPLES of MONEY.

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
7	13	3	14	7	5	15	17	10	53	14	8
3	5	10½	8	19	2¼	3	14	6	5	10	2¼
6	18	7	5	3	4½	23	6	2¼	93	11	6
-	2	5¼	21	2	9	8	3	5	7	5	-
4	-	3	7	16	8½	15	6	4	2	-	9
17	15	4½	-	4	3	6	12	9¼	-	18	7

COMPOUND ADDITION.

21

<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
14	9	7½	37	15	8	61	3	2½	472	15	3
5	13	6	14	12	9½	7	16	8	9	2	2½
62	4	7	5	6	11	29	13	10½	27	12	6½
4	17	8	23	10	9½	8	14	-	370	16	2½
23	-	4½	8	6	-	-	7	5½	25	3	8
6	6	7	14	-	5½	24	13	-	6	10	5½
91	-	10½	54	2	7½	5	-	10½	30	-	11½

Suppose that *A* is indebted to *B*, 34*l.* 13*s.* 7*d.* to *C*, 1730*l.* to *D*, 9*l.* 19*s.* 2*d.* to *E*, 134*l.* 7*d.* to *F*, 17*s.* 2*d.* and to *G*, 9*d.* what is *A*'s whole debt?

Ans. 1909*l.* 11*s.* 3*d.*

Suppose that *B* owes *A* 75*l.* 17*s.* *C* owes 15*s.* 5*d.* *D* owes 21*l.* 13*s.* 6½*d.* *E* owes 9½*d.* *F* owes 796*l.* 3*d.* and *G* owes 17*l.* 13*s.* 10*d.* What is due to *A* by all of them?

Ans. 912*l.* 10*d.*

A owes to *B*, for tea, 13*l.* 10*s.* for cheeses, 17*l.* 13*s.* 5*d.* for cotton, 208*l.* 17*s.* for indian chints, 86*l.* 7*d.* for his acceptance of a bill, 300*l.* for factorage, 15*l.* 17*s.* 3½*d.* also for insurance and other charges, 30*l.* 10*s.* 4½*d.* How much is *A*'s whole debt to *B*?

Ans. 672*l.* 8*s.* 8½*d.*

A corn-factor pays, for wheat, 37*l.* 15*s.* 8*d.* for rye, 11*l.* 16*s.* 3*d.* for oats, 96*l.* 7½*d.* for barley, 53*l.* 12*s.* also for peas and beans, 10*l.* he has also paid for cartilage and other petty charges, 3*l.* 17*s.* 5½*d.* and for insurance, 11*l.* 3½*d.* now suppose that his commission on the whole is 7*l.* 3*s.* 0½*d.* for how much must he draw upon his employer to clear the account?

— 231*l.* 5*s.* 4½*d.*

A nobleman, going out of town, is informed by his steward, that his butcher's bill comes to 197*l.* 13*s.* 7½*d.* his baker's to 59*l.* 5*s.* 2½*d.* his brewer's to 85*l.* his wine-merchant's to 103*l.* 13*s.* to his lordship's corn-chandler is due 75*l.* 3*d.* to his tallow-chandler and cheese-monger, 27*l.* 15*s.* 11½*d.* and to his taylor, 55*l.* 3*s.* 5½*d.* also for rent, servants wages, and other charges,

127*l.*

127*l.* 3*s.* Now supposing he would take 100*l.* with him to defray his charges on the road, for what sum must he send to his banker? — *Ans.* 830*l.* 14*s.* 6½*d.*

EXAMPLES of WEIGHTS, MEASURES, &c.

*Troy Weight**Apothecaries Weight*

lb.	oz.	dwt.	oz.	dwt.	gr.	lb.	oz.	dr.	sc.	oz.	dr.	sc.	gr.
17	3	15	37	9	3	3	5	7	2	3	5	1	17
4	6	3	9	5	13	13	7	3	-	7	3	2	5
-	10	7	3	16	21	9	11	-	1	16	7	-	12
9	5	-	17	7	8	-	9	1	2	9	5	1	5
176	2	17	5	9	-	36	3	5	-	4	1	2	18
23	11	12	3	-	19	5	8	6	1	36	4	1	14

*Avoirdupois Weight**Long Measure*

lb.	oz.	dr.	cwt.	qr.	lb.	mls.	fur.	pls.	yds.	feet	ine.
17	10	13	15	2	15	29	3	14	127	1	5
5	14	8	6	3	24	18	6	29	12	2	9
8	6	15	7	-	10	5	4	20	-	2	6
27	1	9	26	1	17	9	1	37	54	1	11
-	4	-	10	2	6	7	-	3	5	2	7
6	14	10	3	-	3	4	5	9	23	-	5

*Cloth Measure**Land Measure*

yds.	qr.	nl.	el.	en.	qr.	nl.	ac.	ro.	p.	ac.	ro.	p.
26	3	1	270	1	-	225	3	37	19	-	16	
13	1	2	57	4	3	16	1	25	270	3	29	
6	2	-	8	2	1	9	-	13	9	1	3	
217	-	3	-	3	2	4	2	9	23	-	34	
9	1	-	10	1	-	42	1	19	7	2	16	
55	3	1	4	4	1	7	-	6	75	1	23	

Wine

Wine Measure

<i>T.</i>	<i>hds.</i>	<i>gal.</i>	<i>hds.</i>	<i>gal.</i>	<i>pts.</i>
13	3	15	15	61	5
8	1	37	7	16	3
4	2	26	29	23	7
25	-	12	3	15	1
3	1	9	16	8	-
72	3	21	4	36	6

Ale and Beer Measure

<i>hds.</i>	<i>gal.</i>	<i>pts.</i>	<i>hds.</i>	<i>gal.</i>	<i>pts.</i>
17	37	3	29	43	5
4	13	5	7	9	2
3	6	2	14	16	6
5	14	-	6	8	1
12	9	6	57	13	4
8	42	4	5	6	-

Dry Measure

<i>L.</i>	<i>qr.</i>	<i>bu.</i>	<i>qr.</i>	<i>bu.</i>	<i>pe.</i>
13	5	2	25	7	3
7	1	3	5	3	1
41	7	4	17	5	2
3	-	7	6	2	-
24	3	-	33	-	2
5	2	1	7	4	1

Time

<i>mo.</i>	<i>we.</i>	<i>da.</i>	<i>hrs.</i>	<i>m.</i>	<i>s.</i>
17	3	4	27	15	37
26	1	6	12	26	14
7	-	2	3	7	16
19	2	3	35	42	59
8	3	-	6	9	4
12	1	6	31	16	32

A gentleman bought of a silver-smith, dishes to the weight of 23 lb. 6 oz. 5 dwt. plates, 41 lb. 7 oz 17 dwt. spoons, 12 lb. 15 dwt. salts, 2 lb. 7 oz. presenters, 13 lb. and tankards, 7 lb. 17 dwt. What weight of plate did he buy in all? — — *Ans.* 99 lb. 10 oz. 14 dwt.

An apothecary made a composition of 5 ingredients, the 1st of which weigh'd 13 lb. 7 oz. the 2d, 11 oz. 7 dr. 13 gr. the 3d, 7 lb. 2 scr. the 4th, 11 lb. 3 dr. 1 scr. and the 5th weigh'd 15 lb. 5 oz. 7 gr. What was the weight of the whole? — — *Ans.* 48 lb. 3 dr. 1 sc.

A country shop-keeper buys of a merchant in London, teas, weighing 3 qrs. 14 lb. Coffee, 1 qr. 23 lb. sugars, 3 cwt. 2 qr. 5 lb. spices, 2 qr. 3 lb. 13 oz. hops, 13 cwt. 1 qr. 24 lb. and several other things to the weight of 3 cwt. 17 lb.

17 lb. 7 oz. For what weight has he to pay carriage on bringing them home? — *Ans.* 22 cwt. 3 lb. 4 oz.

From *A* to *B* is 3 mls. 2 fur. 7 pls. from *B* to *C* is 17 mls. 13 pls. from *C* to *D* is 7 fur. and from *D* to *E* is 5 mls. 33 pls. What is the distance between *A* and *E*?

Ans. 26 mls. 2 fur. 13 pls.

Bought four parcels of cloth, the 1st of which contains 25 yds. 3 qr. the 2^d, 37 yds. 2 qr. 3 nls. the 3^d, 14 yds. 1 n. and the 4th, 23 yds. How many yards are in them all? — — — — *Ans.* 100 yds. 2 qrs.

There are five pieces of ground, the first of which measures 13 ac. 3 r. 14 p. the second, 27 ac. 29 p. the third, 19 ac. 1 r. the fourth, 3 r. 34 p. and the fifth, 45 ac. 2 r. 11 p. What is the sum of their measures?

Ans. 160 ac. 3 r. 9 p.

A gentleman bought of a wine-merchant, of port wine, 1 tun, 3 hhds. of claret, 3 hhds. 47 gal. of mountain, 1 hhd. 5 gal. and of Lisbon, 2 hhds. 23 gal. What quantity did he buy in all? — — — *Ans.* 3 tn. 2 hhd. 12 gal.

A beer-brewer has sent into the country, ale as follows, viz. at one time, 3 hhds. 14 gal. at another, 2 hhds. 17 gal. at another, 14 hhds. 27 gal. and at another, 5 hhds. 47 gal. How much was sent at all the times?

Ans. 26 hhds. 3 gal.

A corn-merchant sends over the sea, of wheat, 13 lasts, 3 qr. 5 bush. of oats, 29 lfts. 7 qr. of rye, he has sent 3 lfts. 7 bush. of peas, 8 qrs. 3 bush. and of beans, 5 qr. For what has he freight to pay?

Ans. 47 lfts. 4 qr. 7 bush.

When *B* was born, *A*'s age was 113 mths. 2 wks. when *C* was born, *B*'s age was 97 mo. 1 we. 5 ds. when *D* was born, *C*'s age was 107 mo. 3 ds. 14 hrs. and when *E* was born, *D*'s age was 75 mo. 3 we. 19 hrs. What was *A*'s age when *E* was born?

Ans. 393 mo. 3 we. 2 ds. 9 hrs.

COMPOUND SUBTRACTION.

COMPOUND Subtraction is the finding of the difference of two numbers, whereof one or both are compound.

R U L E.

1. Write the less number under the greater, as directed in compound addition.

2. Then, beginning at the least denomination, subtract the under number of each from the upper, writing their respective remainders below them.

3. But if the under number of any of the denominations be greater than the upper, add so many to the upper as make one of the next higher denomination; then take the under from the sum, writing down the remainder as before, and carry or add one to the under number of the next higher denomination before you subtract it.

EXAMPLES of MONEY.

	£.	s.	d.	£.	s.	d.	£.	s.	d.	£.	s.	d.
From	79	17	8½	103	3	2½	57	—	10	25	1	13
Take	35	12	4½	71	12	5½	29	13	3½	35	4	7½
	<hr/>			<hr/>			<hr/>			<hr/>		

Rem.

Proof

What is the difference between 73*l.* 5½*d.* and 19*l.* 13*s.* 10*d.*? — — — *Ans.* 53*l.* 6*s.* 7½*d.*

A lends to *B* 100*l.* how much is *B* in his debt, after *A* has taken goods of him to the amount of 73*l.* 12*s.* 4½*d.* — — — *Ans.* 26*l.* 7*s.* 7½*d.*

Suppose that my rent for half a year is 10*l.* 12*s.* and that I have laid out for the land tax, 14*s.* 6*d.* and for several repairs, 1*l.* 3*s.* 3½*d.* what have I to pay of my half year's rent? — — — *Ans.* 8*l.* 14*s.* 2½*d.*

A trader failing, owes to *A*, 35*l.* 7*s.* 9*d.* to *B*, 91*l.* 13*s.* ½*d.* to *C*, 53*l.* 7½*d.* to *D*, 87*l.* 5*s.* and to *E*,

D

118*l.*

26 COMPOUND SUBTRACTION.

111 *l.* 3 *s.* 5½ *d.* When this happened, he had by him in cash, 23 *l.* 7 *s.* 5 *d.* in wares, 53 *l.* 11 *s.* 10½ *d.* in household furniture, 63 *l.* 17 *s.* 7½ *d.* and in recoverable book-debts, 25 *l.* 7 *s.* 5 *d.* What will his creditors lose by him, supposing these things delivered to them?

Ans. 212 *l.* 5 *s.* 6½ *d.*

EXAMPLES of WEIGHTS, MEASURES, &c.

Troy Weight.

	lb.	oz.	dwt.	gr.	lb.	oz.	dwt.	gr.
From	7	3	14	11	4	9	1	23
Take	3	7	5	19	3	7	16	12
	<hr/>				<hr/>			

Rem.

Proof

Apothecaries Weight.

	lb.	oz.	dr.	scr.	gr.
	73	4	7	—	14
	26	7	2	1	16
	<hr/>				

Avoirdupois Weight.

	C.	qr.	lb.	lb.	oz.	dr.
From	5	—	17	71	5	9
Take	3	2	11	14	6	14
	<hr/>			<hr/>		

Rem.

Proof

Long Measure.

	M.	fu.	pl.	yd.	ft.	in.
	14	3	17	96	1	4
	3	7	9	41	2	7
	<hr/>			<hr/>		

Cloth Measure.

	yd.	qr.	nl.	yd.	qr.	nl.
From	17	2	1	9	—	2.
Take	5	2	3	6	1	2
	<hr/>			<hr/>		

Rem.

Proof

Land Measure.

	ac.	ro.	p.	ac.	ro.	p.
	17	1	14	57	1	16
	9	3	6	24	2	25
	<hr/>			<hr/>		

Wine

COMPOUND MULTIPLICATION. 27

Wine Measure.

	T. hd. gal.	hd. gal. pt.
From	17 2 23	5 — 4
Take	4 3 39	3 2 7
	<u> </u>	<u> </u>
Rem.	<u> </u>	<u> </u>
Proof	<u> </u>	<u> </u>

Ale and Beer Measure.

	hd. gal. pt.	hd. gal. pt.
From	14 29 3	7 16 5
Take	7 34 5	17 3 2
	<u> </u>	<u> </u>
Rem.	<u> </u>	<u> </u>
Proof	<u> </u>	<u> </u>

Dry Measure.

	la. qr. bu.	bu. gal. pt.
From	9 4 7	13 7 1
Take	3 7 2	7 3 4
	<u> </u>	<u> </u>
Rem.	<u> </u>	<u> </u>
Proof	<u> </u>	<u> </u>

Time.

	mo. we. da.	ds. hrs. min.
From	7 12 5	11 4 17 26
Take	14 3 —	75 12 33
	<u> </u>	<u> </u>
Rem.	<u> </u>	<u> </u>
Proof	<u> </u>	<u> </u>

COMPOUND MULTIPLICATION.

COMPOUND Multiplication is the finding of a number which shall contain a given compound number any proposed number of times.

R U L E.

1. Write the multiplier under the lowest denomination of the multiplicand.
2. Multiply the number of the lowest denomination by the multiplier, and find how many ones of the next higher denomination are contained in the product, as in compound addition.
3. Write down the excess, and carry the ones to the product of the next higher denomination, with which proceed as before; and in like manner with all the other denominations to the highest.

I. EXAMPLES of MONEY.

3 lb. of green tea, at 9 s.	4 lb. of bohea tea, at 7 s.
6 d. per lb. <i>Ans.</i> 1 l. 8 s. 6 d.	8 d. — <i>Ans.</i> 1 l. 10 s. 8 d.
5 lb. of sugar, at 1 s. 3 d.	6 lb. of flax, at 9½ d.
<i>Ans.</i> 6 s. 3 d.	<i>Ans.</i> 4 s. 9 d.
7 lb. of tobacco, at 1 s.	8 stones of beef, at 2 s.
8½ d. — <i>Ans.</i> 11 s. 11½ d.	7½ d. — <i>Ans.</i> 1 l. 1 s.
9 lb. of galls, at 1 s. 5 d.	10 cwt. of cheese, at 2 l.
<i>Ans.</i> 12 s. 9 d.	17 s. 10 d.
11 cwt. of cheese, at 1 l.	<i>Ans.</i> 28 l. 18 s. 4 d.
15 s. 6 d.	12 cwt. of sugar, at 3 l.
<i>Ans.</i> 19 l. 10 s. 6 d.	7 s. 4 d. — <i>Ans.</i> 40 l. 8 s.

If the multiplier exceed 12, it is commonly best to multiply successively by its component parts, as in simple multiplication.

E X A M P L E S.

14 moidores, at 1 l. 7 s.	15 pistoles, at 17 s. 6 d.
<i>Ans.</i> 18 l. 8 s.	<i>Ans.</i> 13 l. 2 s. 6 d.
16 cwt. of cheese, at 1 l.	18 cwt. of tobacco, at
18 s. 8 d.	5 l. 11 s. 4 d.
<i>Ans.</i> 30 l. 18 s. 8 d.	<i>Ans.</i> 100 l. 4 s.
20 cwt. of hops, at 4 l.	21 cwt. of hemp, at 1 l.
7 s. 2 d. — <i>Ans.</i> 87 l. 3 s. 4 d.	12 s. — <i>Ans.</i> 33 l. 12 s.
22 tons of hay, at 1 l. 2 s.	24 tons of hay, at 3 l.
<i>Ans.</i> 24 l. 4 s.	7 s. 6 d. — <i>Ans.</i> 81 l.
25 yds. of broad cloth, at	27 yds. of fine broad cloth,
9 s. 2 d. — <i>Ans.</i> 11 l. 9 s. 2 d.	at 15 s. 7 d.
28 yds. superfine broad	<i>Ans.</i> 21 l. 9 d.
cloth, at 19 s. 4 d.	30 yds. of shalloon, at
<i>Ans.</i> 27 l. 1 s. 4 d.	2 s. 2½ d. — <i>Ans.</i> 3 l. 6 s. 3 d.
32 yds. german serge, at	33 yds. of flannel, at 1 s.
3 s. 7 d. — <i>Ans.</i> 5 l. 14 s. 8 d.	3 d. — <i>Ans.</i> 2 l. 1 s. 3 d.
35 yds. irish cloth, at 2 s.	36 yds. of scotch cloth,
5½ d. — <i>Ans.</i> 4 l. 6 s. ½ d.	at 2 s. 9 d. — <i>Ans.</i> 4 l. 19 s.
40 ells of holland, at 5 s.	42 ells of holland, at 6 s.
6 d. — — <i>Ans.</i> 11 l.	9½ d. — <i>Ans.</i> 14 l. 5 s. 3 d.

64 gal-

44 ells of dowlas, at 1s.
4l. — *Ans.* 2l. 18s. 8d.
48 ac. of arable ground,
at 2l. 3s. — *Ans.* 103l. 4s.
50 ac. pasture ground,
at 1l. 4s. 6d. *Ans.* 61l. 5s.
55 tuns of wine, at 83l.
10s. — *Ans.* 4592l. 10s.
60 gallons of wine, at
5s. 8d. — *Ans.* 17l.
64 gallons of brandy, at
9s. 6d. — *Ans.* 30l. 8s.
70 barrels of ale, at 1l.
4s. — — *Ans.* 84l.
77 firkins of beer, at
11s. 7d. *Ans.* 44l. 11s. 11d.
81 firkins of soap, at 1l.
8s. 9d. — *Ans.* 116l. 8s. 9d.
88 qrs. of oats, at 11s.
6d. — *Ans.* 90l. 12s.
96 qrs. of rye, at 1l. 3s.
4d. — — *Ans.* 112l.
100 stones of wool, at
7s. 3d. — *Ans.* 36l. 5s.
120 doz. candles, at 5s.
9d. — *Ans.* 34l. 10s.
132 days wages, at 2s.
4d. — *Ans.* 15l. 8s.

45 ells of dowlas, at 1s.
6d. — *Ans.* 3l. 7s. 6d.
49 acr. meadow, at 1l.
7s. 10. — *Ans.* 68l. 3s. 10d.
54 acr. land, at 1l. 13s.
Ans. 89l. 2s.
56 pipes of wine, at 37l.
7s. 6d. — *Ans.* 2093l.
63 gall. of oil, at 2s. 3d.
Ans. 7l. 1s. 9d.
66 gall. of rum, at 8s.
10d. — *Ans.* 29l. 3s.
72 bds. at 1l. 14s. 4d.
Ans. 123l. 12s.
80 firkins of butter, at
1l. 5s. 6d. — *Ans.* 102l.
84 qrs. of wheat, at 1l.
12s. 8d. — *Ans.* 137l. 4s.
90 qrs. of barley, at 17s.
10d. — *Ans.* 80l. 5s.
99 bushels of malt, at
6s. 3½d. *Ans.* 31l. 2s. 10½d.
110 sheep, at 12s. 8d.
Ans. 69l. 13s. 4d.
121 weeks wages, at 7s.
6d. — *Ans.* 45l. 7s. 6d.
144 reams of paper, at
13s. 4d. — *Ans.* 96l.

+ Note, The following examples require the continual product of three numbers.

112 lb. or 1 cwt. at 3s.
4½d. per lb. — *Ans.* 18l. 18s.
224 lb. or 2 cwt. at 7s.
3½d. per lb.
Ans. 81l. 8s. 8d.
336, at 1s. 5d. each.
Ans. 23l. 16s.

1 cwt. at 1s. 9d. per lb.
Ans. 9 l. 16s.
336 lb. or 3 cwt. at 5s.
2½d. per lb.
Ans. 87l. 17s.
350, at 3s. 2½d. each.
Ans. 55l. 15s. 7½d.

30 COMPOUND MULTIPLICATION.

But if the multiplier cannot be produced by the multiplication of small numbers, find the nearest to it, either greater or less, which can be so produced; then after having multiplied continually by the component parts of this number, to or from the last product, add or subtract the produce of so many as it is less, or greater than the given number.

EXAMPLES.

17 at 5s. 6d.
Ans. 4l. 13s. 6d.

23 at 1s. 6½d.
Ans. 1l. 15s. 5½d.

29 at 2l. 5s. 3¼d.
Ans. 65l. 12s. 10¼d.

34 at 19s. 7d.
Ans. 33l. 5s. 10d.

38 at 1l. 11s. 5½d.
Ans. 99l. 15s. 5d.

41 at 3s. 1d.
Ans. 6l. 6s. 5d.

46 at 4s. 7¼d.
Ans. 10l. 11s. 9½d.

51 at 6s. 7¼d.
Ans. 16l. 18s. 11¼d.

59 at 7s. 10d.
Ans. 23l. 2s. 2d.

68 at 9s. 11¼d.
Ans. 33l. 15s. 9d.

79 at 11s. 5½d.
Ans. 45l. 6s. 10¼d.

94 at 12s. 2d.
Ans. 57l. 3s. 8d.

106 at 14s. 7¼d.
Ans. 77l. 8s. ½d.

117 at 1l. 2s. 3d.
Ans. 130l. 3s. 3d.

19 at 13s. 2d.
Ans. 12l. 10s. 2d.

26 at 3s. ¼d.
Ans. 3l. 19s. 7½d.

31 at 17s. 5½d.
Ans. 27l. 1s. 2½d.

37 at 12s. 10¼d.
Ans. 23l. 15s. 7¼d.

39 at 7s. 3½d.
Ans. 14l. 5s. 2¼d.

43 at 2s. 10d.
Ans. 6l. 1s. 10d.

47 at 5s. 2½d.
Ans. 12l. 4s. 9½d.

53 at 3l. 15s. 2d.
Ans. 199l. 3s. 10d.

62 at 8s. 5d.
Ans. 26l. 1s. 10d.

74 at 10s. ½d.
Ans. 37l. 3s. 1d.

86 at 7d.
Ans. 2l. 10s. 2d.

104 at 13s. 5½d.
Ans. 69l. 19s. 8d.

114 at 15s. 3¼d.
Ans. 87l. 5s. 7½.

127 at 3l. 2d.
Ans. 382l. 1s. 2d.

II. EXAMPLES of WEIGHTS, MEASURES, &c.

lb.	oz.	dwt.	gr.
3	—	14	9
			3

cwt.	qr.	lb.
17	3	23
		5

Miles	fur.	poles
31	3	27
		7

yd.	qr.	nl.
53	1	3
		9

Tuns	hds.	gal.
19	3	17
		11

Hhd fir. pts.

Mult. 3 5 47 by 18.

Ans. 71 hds. 30 pints.
las. qr. bus.

Mult. 5 3 7 by 72.

Ans. 387 las. 9 qrs.

Mon. we. da.

Mult. 7 3 5 by 26.

Ans. 206 mo. 4 da.

cwt. qr. lb.

Mult. 3 7 14 by 53.

Ans. 178 cwt. 3 qr. 14 lb.

lb.	oz.	dr.	sc.	gr.
13	5	3	1	14
				4

lb.	oz.	dr.
21	11	15
		6

yd.	feet	inc.
171	1	10
		8

ac.	ro.	pol.
15	2	29
		10

Hds.	gal.	pin.
53	33	3
		12

Bar. kild. gal.

Mult. 21 1 13 by 36.

Ans. 787 bar. 1 kil. 9 gal.

Bus. pec. gal.

Mult. 71 3 1 by 132.

Ans. 9487 bu. 2 pec.

Days hrs. min.

Mult. 14 13 27 by 47.

Ans. 684 da. 8 hr. 9 m.

cwt. lb.

Mult. 17 — 12 By 75.

Ans. 1283 cwt. 4 lb.

COMPOUND DIVISION.

COMPOUND Division is the dividing either a simple or compound number into any proposed number of equal parts, whereof each shall be a compound number.

R U L E.

1. Place the divisor and dividend as in simple division.
2. Begin at the highest denomination and divide each by the divisor, writing the quotes under their respective dividends.
3. But if there be a remainder, after dividing any of the denominations except the least, you must find how many of the next lower denomination it is equal to, and add to it the small number (if any) which was in this denomination before; then divide the sum.

I EXAMPLES of MONEY.

$$\begin{array}{r} \text{l. s. d.} \\ 3) \quad 1 \quad 8 \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{s. d.} \\ 5) \quad 6 \quad 3 \\ \hline \end{array}$$

$$\begin{array}{r} \text{s. d.} \\ 7) \quad 11 \quad 11\frac{1}{2} \\ \hline \end{array}$$

$$\begin{array}{r} \text{s. d.} \\ 9) \quad 12 \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{l. s. d.} \\ 11) \quad 19 \quad 10 \quad 6 \\ \hline \end{array}$$

$$\begin{array}{r} \text{l. s. d.} \\ 4) \quad 1 \quad 10 \quad 8 \\ \hline \end{array}$$

$$\begin{array}{r} \text{s. d.} \\ 6) \quad 4 \quad 9 \\ \hline \end{array}$$

$$\begin{array}{r} \text{l. s.} \\ 8) \quad 1 \quad 1 \\ \hline \end{array}$$

$$\begin{array}{r} \text{l. s. d.} \\ 10) \quad 28 \quad 18 \quad 4 \\ \hline \end{array}$$

$$\begin{array}{r} \text{l. s.} \\ 12) \quad 40 \quad 8 \\ \hline \end{array}$$

If the divisor exceed 12, it is best to divide continually by its component parts, as in simple division.

What is cheese *per cwt.* if
16 *cwt.* cost 30*l.* 18*s.* 3*d.*?

Ans. 1*l.* 18*s.* 3*d.*

If 22 cost 24*l.* 4*s.* what
is 1?

Ans. 1*l.* 2*s.*

Divide 5*l.* 14*s.* 8*d.* by
32. — *Quot.* 3*s.* 7*d.*

Divide 2*l.* 18*s.* 8*d.* into
44 equal parts.

Ans. each part is 1*s.* 4*d.*

Divide 4592*l.* 10*s.* e-
qually among 55 persons.

Ans. each one's share is
83*l.* 10*s.*

If 70 be 84*l.* what is 1?

Ans. 1*l.* 4*s.*

If 88 cost 50*l.* 12*s.* what
will 1 be?

Ans. 1*l.* 6*d.*

Divide 34*l.* 10*s.* by 120.

Quot. 5*s.* 9*d.*

What is tobacco *per cwt.*
if 18 *cwt.* cost 100*l.* 4*s.*

Ans. 5*l.* 11*s.* 4*d.*

If 27 cost 21*l.* 9*d.* what
is 1? — *Ans.* 15*s.* 7*d.*

Divide 4*l.* 19*s.* by 36.

Quot. 2*s.* 9*d.*

Divide 68*l.* 3*s.* 10*d.* in-
to 49 equal parts.

Ans. each part is 1*l.* 7*s.*
10*d.*

Divide 7*l.* 1*s.* 9*d.* equal-
ly among 63 persons.

Ans. each one's share is
2*s.* 3*d.*

If 80 be 102*l.* what is 1?

Ans. 1*l.* 5*s.* 6*d.*

If 99 cost 31*l.* 2*s.* 10*d.*
what will 1 be?

Ans. 6*s.* 3½*d.*

Divide 45*l.* 7*s.* 6*d.* by
121 — *Quot.* 7*s.* 6*d.*

Note, The following examples require three divisions.

At 18*l.* 18*s.* *per cwt.*
how much *per lb.*

Ans. 3*s.* 4½*d.*

Divide 81*l.* 8*s.* 8*d.* by
224 — *Quot.* 7*s.* 3½*d.*

At 9*l.* 16*s.* *per cwt.*
how much *per lb.*?

Ans. 1*s.* 6*d.*

Divide 55*l.* 15*s.* 7½*d.* by
350. — *Quot.* 3*s.* 2½*d.*

But if the divisor cannot be produced by the multipli-
cation of small numbers, you must divide by it after the
manner of long division.

EXAMPLES.

	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>		
17)	4	13	6	(5	6	19)	12	10	2	(13	2		
	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>	
29)	65	12	10½	(2	5	3½	37)	23	15	7½	(12	10½	
	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>			<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
41)	6	6	5	(3	1	53)	199	3	10	(3	15	2	
	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>s.</i>	<i>d.</i>			<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>l.</i>	<i>s.</i>	<i>d.</i>
86)	2	10	2	(7		127)	382	1	2	(3	2		

II. EXAMPLES of WEIGHTS, MEASURES, &c.

	lb.	oz.	dwt.		lb.	oz.	dr.	sc	gr.
4)	13	1	15(6)	2	5	3	— 10(
	C.	qr.	lb.			lb.	oz.	dr.	
7)	75	1	12(9)	5	3	14(
	Miles	fur.	pls.			yd.	feet	in.	
11)	58	5	12(12)	150	1	7(
	yd.	qr.	nls.			acr.	ro.	pls.	
17)	31	2	3(26)	17	3	17(

RULE-OF-THREE.

THE Rule-of-Three is that by which a number is found having to a given number the same proportion which is between two other given numbers. For this reason it is sometimes named the *rule of proportion*.

It is called the rule-of-three, because in each of its questions there are given three numbers at least. And because of its excellent and extensive use, it is often named the *golden rule*.

For the stating, or rightly placing down the three given numbers, observe the following

R U L E.

1. Write down the number which is of the same kind with the answer or number required.

2. Consider whether the answer ought to be greater or less than this number : then write respectively the greater or

or less of the two remaining numbers on the right of it for the third, and the other on the left for the first number or term.

Multiply the 2d and 3d terms together, divide the product by the 1st, and the quotient will be the answer.

Note 1. When you can conveniently multiply and divide as in compound multiplication and division, it is best so to do.

2. But if not, reduce the compound terms to the lowest name mentioned in them, and the 1st and 3d to the same name if they be not so already; then will the answer be of the same name with the 2d term.

3. When there happens to be a remainder after division, reduce it to the name next below the last quotient, and divide by the same divisor; so shall the quote be so many of the said next name; do this so long as there is any remainder, till you have reduced it to the least name, and all the quotients together will be the answer.

4. If the 1st term, and either the 2d or 3d can be divided by any number, without remainder; let them be divided, and the quotients used instead of them.

5. There are four other methods of operation besides the general one above delivered, any of which, when possible, performs the work much shorter than it. They are thus:

First, Divide the 2d term by the first, multiply the quotient into the 3d, and the product will be the answer.

Second, Divide the 3d term by the 1st, multiply the quotient into the 2d, and the product will be the answer.

Third, Divide the 1st term by the 2d, divide the 3d by the quotient, and the last quotient will be the answer.

Fourth, Divide the 1st term by the 3d, divide the 2d by the quotient, and the last quotient will be the answer.

E X A M P L E S

1. If 8 yards of cloth cost 24s. what will 96 yards cost?

Ans. 14l. 8s.

2. How many yards of cloth may be bought for 14l. 8s. when 8 yards cost 24s? — *Ans.* 96 yds.

3. What will be the price of 72 yards of cambric, of which 9 yards cost 5l. 12s? — *Ans.* 44l. 16s.

4. What will 9 yards of cambric cost, at the rate of 44l. 16s. for 72 yards? — — *Ans.* 5l. 12s.

5. If 7 cwt. 1 qr. of sugar, cost 26l. 10s. 4d. what will be the price of 43 cwt. 2 qrs? — *Ans.* 159l. 2s.

6. What quantity of sugar may be bought for 159l. 2s. at the rate of 7 cwt. 1 qr. for 26l. 10s. 4d.

Ans. 43 cwt. 2 qrs.

7. How

7. How many yards of muslin may be bought for 44*l*. 16*s*. whereof 9 yards cost 5*l*. 12*s*? — *Ans*. 72 yards.

8. How much must be paid for 26 bags of hops, at 21*l*. 4*s*. for 8 bags? — *Ans*. 68*l*. 18*s*.

9. How many men must be employed to finish a piece of work in 15 days, which 5 men can do in 24 days?

Ans. 8 men.

10. How many yards of broad-cloth may be bought for 5*l*. 12*s* of which 72 yards cost 44*l*. 16*s*. — *Ans*. 9 yds.

11. What must be paid for 53 *ells* *eng*. 1*qr*. of holland at the rate of 7*s*. 9½*d*. per yard? — *Ans*. 25*l*. 18*s*. 1½*d*.

12. How many yards of matting, which is 2 feet 6 inches broad, will cover a floor which is 27 feet long and 20 broad? — *Ans*. 216 feet or 72 yds.

13. What quantity of sugar may be bought for 26*l*. 10*s*. 4*d*. when the price of 43 *cwt*. 2 *qr*. is 150*l*. 2*s*?

Ans. 7 *cwt*. 1 *qr*.

14. In how many days will 8 men finish a piece of work which 5 men can do in 24 days? — *Ans*. 15 days.

15. What must be paid for 8 bags of hops, when the price of 26 bags is 68*l*. 18*s*? — *Ans*. 21*l*. 4*s*.

16. A person breaking, owes in all, 977*l*. and has in money, goods, and recoverable debts, 420*l*. 6*s*. 3½*d*. supposing these things delivered to his creditors, what will they get per pound? — *Ans*. 8*s*. 7½*d*.

17. What must be given for a piece of silver weighing 73*lb*. 5*oz*. 15 *dwt*s. at the rate of 5*s*. 9*d*. per ounce?

Ans. 253*l*. 10*s*. ¾*d*.

18. Bought 3 casks of raisins, each weighing 3 *cwt*. 1 *qr*. 7*lb*. neat, what will they cost at 2*l*. 6*s*. 6*d*. per *cwt*? — *Ans*. 23*l*. 2*s*. 1½*d*.

19. A garrison being besieged, has 5 months provision in it, at the rate of 12 ounces a day for each man; but being informed that it cannot be relieved till after 9 months, how much per day must each man have that the provisions may last that time? — *Ans*. 6½ ounces.

20. What will the tax upon 763*l*. 15*s*. be, at the rate of 3*s*. 6*d*. per pound? — *Ans*. 133*l*. 13*s*. 1½*d*.

21. How

21. How much silver may I have for 253*l*. 10*s*. 0*d*. at 5*s*. 9*d*. per ounce? — *Ans*. 73*lb*. 5*oz*. 15*dwt*s.
22. What will 7*cwt*. 1*qr*. of sugar cost, at the rate of 43*cwt*. 2*qr*. for 159*l*. 2*s*? — *Ans*. 26*l*. 10*s*. 4*d*.
23. What must be paid for 1*cwt*. 3*qr*. 17*lb*. of wool, at 7*s*. 4*d*. the stone of 14*lb*? — *Ans*. 5*l*. 11*s*. 6*d*. 3*q*.
24. What quantity of hops may be bought for 68*l*. 18*s*. of which 8 bags cost 21*l*. 4*s*? — *Ans*. 26 bags.
25. How many ells eng. of holland may be bought for 25*l*. 18*s*. 1*d*. at 7*s*. 9*d*. per yard? *Ans*. 53 ells eng. 1*qr*.
26. A person breaking, owes to several 977*l*. but compounds with them for 8*s*. 7*d*. per pound, what must he pay them in all? — — *Ans*. 420*l*. 6*s*. 3*d*.
27. A borrowed of B 730*l*. for 8 months; afterwards A would requite B's kindness by lending him 375*l*. required the time it must be lent. *Ans*. 15 mo. 2 we. 2*d*.
28. What will 1*qr*. 1 nail of velvet cost at 18*s*. 6*d*. per yard? — — *Ans*. 5*s*. 9*d*. 1*q*.
29. What must be given for 7*c*. 3*qr*. 14*lb*. of cheese, at 1*l*. 14*s*. 2*d*. per *cwt*? — — *Ans*. 13*l*. 9*s*. 0*d*.
30. What is the price of 2*c*. 1*qr*. 12*lb*. of beef, at 2*s*. 8*d*. per stone of 14*lb*? — *Ans*. 2*l*. 10*s*. 3*d*. 1*q*.
31. A person breaking, compounds with his creditors for 8*s*. 7*d*. per pound, and at that rate he pays them in all 420*l*. 6*s*. 3*d*. What was his debt? — *Ans*. 977*l*.
32. Bought 73*lb*. 5*oz*. 15*dwt*s. of silver for 253*l*. 10*s*. 0*d*. What did it cost per ounce? — *Ans*. 5*s*. 9*d*.
33. If the tax upon 763*l*. 15*s*. be 133*l*. 13*s*. 1*d*. at what rate is it per pound. — — *Ans*. 3*s*. 6*d*. per *l*.
34. What must I pay for three-eighths of a ship, which is valued at 700*l*. — — *Ans*. 262*l*. 10*s*.
35. What will the carriage of 8*c*. 3*qrs*. 7*lb*. cost at 10*d*. per stone? — — *Ans*. 2*l*. 18*s*. 9*d*.
36. If the carriage of 3*c*. 14*lb*. for 96 miles be 1*l*. 12*s*. 6*d*. how far may I have 3*c*. 1*qr*. carried for the same money? — — *Ans*. 151 m. 3 fur. 3*1*/*4* poles.
37. What cost 30 pieces of lead, each weighing 1*c*. 12*lb*. at the rate of 16*s*. 4*d*. per *cwt*? *Ans*. 27*l*. 2*s*. 6*d*.

38. Bought a silver tankard, weighing 1 lb. 7 oz. 14 dwt. what will it cost me at 6s. 4d. the ounce?

Ans. 6l. 4s. 9½d.

39. What must be paid for 7 casks of prunes, each weighing 2 c. 1 qr. 14 lb. at 2l. 19s. 8d. per cwt?

Ans. 49l. 11s. 11½d.

40. Bought 14 pockets of hops, each weighing 1 c. 1 qr. 18 lb. at 4l. 2s. 6d. per cwt. what do they come to?

Ans. 81l. 9s. 4½d.

41. What must be given for 75 chaldrons, 7 bushels of coals, at the rate of 1l. 13s. 6d. per chaldron?

Ans. 125l. 9s. 0½d.

42. How much must be paid for 17 qrs. 1 peck of corn, at 3s. 10d. per bushel?

Ans. 26l. 2s. 3½d.

43. What cost 43 qrs. 5 bush. of corn, at 1l. 8s. 6d. the quarter?

Ans. 62l. 3s. 3¼d.

44. How much a year will 173 acres, 2 ro. 14 pls. of land give at the rate of 1l. 7s. 8d. per acre?

Ans. 240l. 2s. 7½d.

45. What will 139 pigs of lead, weighing in all 243 cwt. 2 qrs. come to, at 9l. 18s. per fother of 21 cwt?

Ans. 114l. 15s. 10d. 1¼q.

46. What must be paid for 73 pieces of lead, each weighing 1 c. 3 qr. 7 lb. at 10l. 4s. per fother of 19½ cwt?

Ans. 69l. 4s. 2d. 1¼q.

47. If 5 yards of cloth cost 14s. 2d. what must be given for 9 pieces containing each 21 yds. 1 qr?

Ans. 27l. 1s. 10½d.

48. If a gentleman's Estate be worth 2107l. 12s. a year, what may he spend a day to save 500l. in the year?

Ans. 4l. 8s. 1½d.

RULE-OF-FIVE.

THIS rule is so called because that in it there are five numbers or terms given to find a sixth.—It is often named the *double rule-of-three*, because its questions are sometimes performed by two operations of the rule-of-three,

Note,

Note.—Of the five given numbers, three contain a supposition, and the other two a demand; one of the terms of supposition being of the same kind with the number required, and the other two of the same kind as the demanding terms.

RULE for STATING.

1. Write down the term of supposition which is of the same kind as the answer, for the middle term.

2. Take one of the other two terms of supposition, and of the demanding terms, both of a kind; and from the directions given in the *rule-of-three*, consider which places they would possess if a stating were made of them and the middle term only, and place them accordingly; do the same with the other term of supposition and its correspondent demanding one, writing the terms under each other which fall on the right and left of the middle term.

METHOD of OPERATION.

1. *By two operations.*—Take the two upper terms and the middle term in the same order as they stand, for the first stating of the rule-of-three: then take the fourth number resulting from the first stating, for the middle term, and the two under terms in the general stating, in the same order as they stand, for the extreme terms of the second stating; and the fourth term resulting from it will be the answer.

2. *By one operation.*—Multiply together the terms of which the one is above the other, on both sides of the middle term; then account the two products and the middle term as they stand, the three terms of a rule-of-three stating, and the fourth term thence resulting will be the answer.

EXAMPLES.

1. If 100*l* in one year gain 5*l*. interest, what will be the Interest of 750*l*. for 7 years? — *Ans.* 262*l*. 10*s*.

2. If 27*s* be the wages of 4 men for 7 days, what will be the wages of 14 men for 10 days? — *Ans.* 6*l*. 15*s*.

3. What principal will gain 262*l*. 10*s*. in 7 years, at 5*l*. *per cent. per Annum*? — — *Ans.* 750*l*.

4. If a Footman travel 130 miles in 3 days when the days are 12 hours long, in how many days of 10 hours each, may he travel 360 miles? — *Ans.* 94*7* days.

5. A wall which was to be built to the height of 27 feet, was raised to the height of 9 feet by 12 men in 6 days; how many men must be employed to finish the wall in 4 days, at the same rate of working? *Ans.* 36 men.

6. If the price of 10 ounces of bread, when the corn is at 4s. 3d. per bushel, be $3\frac{1}{4}$ d. what must be paid for 2 lb. 3 oz. when the corn is 5s. per bushel? — *Ans.* $11\frac{3}{4}$ d.

7. What is the interest of 340l. for $2\frac{1}{2}$ years, at $4\frac{1}{2}$ l. per cent. per ann. — — — *Ans.* 38l. 5s.

8. If 120 bushels of corn can serve 14 horses 56 days, how many days will 94 bushels serve 6 horses?

Ans. $102\frac{1}{4}$ days.

9. If 7 oz. 5 dwts. of bread be bought for $4\frac{1}{4}$ d. when corn is at 4s. 2d. per bushel, what weight of it may be bought for 1s. 2d. when the price of the bushel is 5s. 6d?

Ans. 1 lb. 4 oz. $3\frac{7}{8}$ dwts.

10. If the carriage of 13 cwt. 1 qr. for 72 miles be 2l. 10s. 6d. what will be the carriage of 7 cwt. 3 qrs for 112 miles? — — — *Ans.* 2l. 5s. 11d. $1\frac{7}{7}$ q.

11. What is the interest of 300l. for 5 weeks, at 5l. per cent. per ann? — — — *Ans.* 1l. 8s. $10\frac{3}{4}$ d.

12. If 3000 lb. of beef serve 340 seamen 15 days, how many lb. will serve 120 seamen 25 days?

Ans. 1764 lb. $11\frac{1}{4}$ oz.

VULGAR FRACTIONS.

A FRACTION, or broken number, is an expression of one or more parts of any number.

The number of parts into which the number is supposed to be divided, is called the *denominator*; and the number of those parts expressed by the fraction, is called the *numerator*.—Also these two numbers are in general named the *terms* of the fraction.

If the number of which the fraction is part, or parts, be 1, it is called a *simple* fraction; and is denoted by the numerator wrote above the denominator with a small line between them: So, $\frac{1}{4}$, denotes one-fourth of 1; $\frac{3}{5}$, denotes three-fifths of 1.

But

But if the number be different from 1, the fraction is called a *compound* one, and is denoted by the word *of*, and the number subjoined to the numerator and denominator expressed as before: So, $\frac{1}{4}$ of 6, denotes one-fourth of 6; $\frac{3}{5}$ of 8, denotes three-fifths of 8; and $\frac{2}{3}$ of $\frac{3}{4}$, denotes two-thirds of three-fourths of 1.

Simple fractions, whose numerators are less than their denominators, are called *proper* fractions.—And those whose numerators are equal to or greater than their denominators, are called *improper* fractions.

The expression formed from an integer and a fraction joined together, is called a *mixt number*.

Note 1. Simple fractions whose numerators are less than, equal to, or greater than their denominators, are respectively less than, equal to, or greater than 1.

2. A fraction, having a fraction or mixt number for its numerator, or denominator, or both, is by some, called a *Complex fraction*.

3. A whole or integer number may be expressed like a fraction by writing 1 under it for a denominator: So, 3 may be denoted by $\frac{3}{1}$ and 12 by $\frac{12}{1}$.

$$4. \left\{ \begin{array}{c} = \\ + \\ - \\ \times \\ \div \end{array} \right\} \text{denotes} \left\{ \begin{array}{c} \text{equ.} \\ \text{add.} \\ \text{sub.} \\ \text{mul.} \\ \text{div.} \end{array} \right\} \text{and is termed} \left\{ \begin{array}{c} \text{is equal to} \\ \text{plus, or more} \\ \text{minus, or less} \\ \text{into} \\ \text{by} \end{array} \right\} \text{thus} \left\{ \begin{array}{l} 8 + 2 = 10 \\ 8 - 2 = 6 \\ 8 \times 2 = 16 \\ 8 \div 2 = 4 \end{array} \right.$$

Besides these, ∞ is wrote between two numbers to denote their difference when it does not appear whether of them is the greater; as $\frac{1}{17} \infty \frac{2}{101}$ denotes the difference of these two fractions.

REDUCTION of VULGAR FRACTIONS.

I. **T**O abbreviate, or reduce fractions to less terms.

RULE I.

Divide the terms of the given fraction by any number which will divide them without a remainder, so shall the quotients be the terms of a new fraction, equal in value to the former; and this you may abbreviate again, and the next again, and so on, till it appear that there is no number greater than 1 which will divide them, in which case the fraction is said to be in its *least* terms.

42 REDUCTION of VULGAR FRACTIONS.

EXAMPLES.

Let $\frac{4}{2}$ be proposed to be abbreviated.

$$\frac{4}{2} = \frac{2}{1} = \frac{2}{1}.$$

Reduce $\frac{3}{4}$ to its least terms.

Reduce $\frac{1}{2}$ to its least terms.

Reduce $\frac{1}{2}$ to its least terms.

Note 1. Any number ending with an even number, or a cypher, may be divided by 2.

EXAMPLES.

$$\frac{2}{4} = \frac{1}{2} = \frac{1}{2}.$$

$$\frac{3}{6} =$$

$$\frac{1}{2} =$$

2. Any number ending with 5 or 0 is divisible by 5.

EXAMPLES.

$$\frac{10}{20} = \frac{1}{2}.$$

$$\frac{75}{150} =$$

3. Any number is divisible by 3 if the sum of its digits be so: Thus 417 is divisible by 3, because 4+1+7, which is the sum of 4, 1, and 7, is 12.

EXAMPLES.

$$\frac{45}{15} = \frac{3}{1} = 3.$$

$$\frac{12}{41} =$$

$$\frac{12}{62} =$$

4. If there be any cyphers at the end of each, cut off so many as are common to both.

EXAMPLES.

$$\frac{200}{300} = \frac{2}{3} = \frac{2}{3}.$$

$$\frac{1200}{18000} =$$

5. When any number, which is expressed by several others with the sign of addition or subtraction between them, is to be divided by any number, then all the parts of it must be divided by this number.

$$\text{Thus } \frac{4+6-8}{2} = 2+3-4 = 5-4 = 1.$$

6. But if the given number be expressed by others with the sign of multiplication between them, only one of them must be divided: So

$$\frac{3 \times 8 \times 10}{2 \times 6} = \frac{3 \times 4 \times 10}{1 \times 6} = \frac{1 \times 4 \times 10}{1 \times 3} = \frac{1 \times 2 \times 10}{1 \times 1} = \frac{20}{1} = 20.$$

And, in this case, when the same number is in both the numerator and denominator, it may be left out of them.

EX-

EXAMPLES.

$$\frac{2 \times 7 \times 9}{3 \times 5 \times 14} =$$

$$\frac{5 \times 2 \times 6}{3 \times 5 \times 2} =$$

$$\frac{7 \times 18 \times 40 \times 9}{10 \times 9 \times 7 \times 6} =$$

RULE 2.

If the fraction must be brought to its least terms at one division; divide its terms by their greatest common-measure, which common-measure is found by dividing the greater term by the less, and this divisor by the remainder; and so on, always dividing the last divisor by the last remainder, till 0 remain; then is the last divisor the greatest common-measure required.

EXAMPLES.

Reduce $\frac{246}{372}$ to its least terms at one division.

First, $246)372(1$

$$\begin{array}{r} \underline{126} \\ 126)246(1 \end{array}$$

$$\begin{array}{r} \underline{120} \\ 120)126(1 \end{array}$$

common-measure 6)120(20

$$\begin{array}{r} 246 \div 6 \quad 41 \\ \text{Then } \frac{\quad}{372 \div 6} = \frac{\quad}{62} \end{array}$$

Reduce $\frac{748}{918}$ to its least terms.

Reduce $\frac{514}{518}$ to its least terms.

Reduce $\frac{5210}{3718}$ to its least terms.

II. To reduce an improper fraction to its equivalent whole or mixt number.

RULE.

Divide the numerator by the denominator, and the quotient will be the integer or mixt number required.

EXAMPLES.

$$\frac{12}{3} = 4.$$

$$\frac{14}{5} =$$

$$\frac{15}{7} = 2 \frac{1}{7}.$$

$$\frac{257}{11} =$$

III. To reduce an integer to an equivalent fraction of a given denomination.

RULE.

R U L E.

Multiply the integer by the given denominator, and the product will be the numerator required.

E X A M P L E S.

Reduce 7 to a fraction whose denominator shall be 4.

$$7 = \frac{7 \times 4}{4} = \frac{28}{4}.$$

Reduce 5 to a fraction whose denominator shall be 9.

Reduce 13 to a fraction whose denominator shall be 12.

IV. *To reduce a mixt number to an equivalent improper fraction.*

R U L E.

Multiply the integer by the denominator of the fraction, to the product add the numerator ; then the sum wrote above the denominator will form the fraction required.

E X A M P L E S.

Reduce $2\frac{3}{7}$ to a fraction.

$$2\frac{3}{7} = \frac{2 \times 7 + 3}{7} = \frac{14 + 3}{7} = \frac{17}{7}.$$

Reduce $12\frac{7}{9}$ to a fraction.

Reduce $14\frac{7}{10}$ to a fraction.

V. *To reduce a compound fraction to an equivalent simple one.*

R U L E.

Multiply all the numerators together for the numerator, and all the denominators together for the denominator of the simple fraction required.

Note, If part of the compound fraction be an integer or a mixt number, reduce it to a fraction by one of the former cases.

E X A M P L E S.

Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of 5 to a simple fraction.

$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{1} = \frac{1 \times 2 \times 3 \times 5}{2 \times 3 \times 4 \times 1} =$ (by omitting the common terms 1, 2, & 3) $\frac{5}{4}$.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $3\frac{1}{2}$ to a simple fraction.

Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{7}{8}$ of 4 to a simple fraction.

VL To reduce fractions of different denominations to equivalent fractions of a common one.

R U L E.

Multiply each numerator continually into all the denominators except its own, for each new numerator; and multiply all the denominators together for the common denominator.

N B. It is evident, that in this, and several other operations, when any of the proposed quantities are integers, mixt numbers, or compound fractions, they must first be reduced by their proper rules to the form of simple fractions.

E X A M P L E S.

Reduce $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$ to a common denomination.

$$\frac{1}{2}, \frac{3}{4}, \text{ and } \frac{5}{8} = \frac{1 \times 1 \times 4}{2 \times 1 \times 4}, \frac{3 \times 2 \times 4}{2 \times 1 \times 4}, \text{ and } \frac{5 \times 2 \times 1}{2 \times 1 \times 4} = \frac{1 \times 2}{4}, \frac{1 \times 6}{4}, \text{ and } \frac{1 \times 8}{4}$$

Reduce $\frac{2}{7}$ and $\frac{5}{9}$ to a common denomination.

Reduce $\frac{1}{7}$, $\frac{3}{8}$, and $5\frac{1}{8}$ to fractions of a com. denom.

Reduce $\frac{5}{8}$, $2\frac{1}{5}$ and 4 to fractions of a com. denom.

Note 1. If the denominators of the given fractions have a common measure, conceive them to be divided by it; then multiply the terms of each given fraction by the quotients of all the denominators, and you will thereby have the new fractions in lower terms than by the general rule.

E X A M P L E S.

Reduce $\frac{1}{12}$, $\frac{7}{8}$, and $\frac{5}{14}$ to a common denomination.

The common measure of these denominators being 4, the quotes are 3, 2, and 6; then

$$\frac{1}{12}, \frac{7}{8}, \text{ and } \frac{5}{14} = \frac{1 \times 2 \times 6}{12 \times 6 \times 2}, \frac{7 \times 3 \times 6}{8 \times 3 \times 6}, \frac{5 \times 3 \times 2}{14 \times 3 \times 2} = \frac{1 \times 2}{12 \times 2}, \frac{7 \times 3}{8 \times 3}, \text{ and } \frac{5 \times 3}{14 \times 3} = \frac{1 \times 2}{12 \times 2}, \frac{7 \times 3}{8 \times 3}, \text{ and } \frac{5 \times 3}{14 \times 3}$$

Reduce $\frac{5}{12}$, $\frac{5}{14}$, and $\frac{1}{18}$ to a common denominator.

Reduce $\frac{5}{8}$, $\frac{3}{12}$, and $\frac{7}{18}$ to a common denomination.

2. Wherefore, if there be only two fractions proposed, multiply the terms of each by the quotient of the denominator of the other.

Reduce $\frac{7}{9}$ and $\frac{4}{15}$ to a common denomination.

$$\frac{7}{9} \text{ and } \frac{4}{15} = \frac{7 \times 5}{9 \times 5} \text{ and } \frac{4 \times 3}{15 \times 3} = \frac{35}{45} \text{ and } \frac{12}{45}$$

Reduce $\frac{2}{15}$ and $\frac{3}{10}$ to a common denomination.

Reduce $\frac{5}{14}$ and $\frac{4}{15}$ to a common denominator.

46 REDUCTION of VULGAR FRACTIONS.

3. If the greatest denominator be divisible by all the rest of the denominators, multiply the terms of all the other fractions by the respective quotients.

EXAMPLES.

Reduce $\frac{1}{11}$, $\frac{7}{8}$, and $\frac{5}{14}$ to a common denominator.

$$\frac{1}{11}, \frac{7}{8}, \text{ and } \frac{5}{14} = \frac{1 \times 2}{11 \times 2}, \frac{7 \times 2}{8 \times 2}, \text{ and } \frac{5}{14} = \frac{5}{14}, \frac{25}{14}, \text{ and } \frac{5}{14}.$$

Reduce $\frac{5}{7}$, $\frac{5}{14}$, and $\frac{9}{8}$ to a common denominator.

Reduce $\frac{5}{8}$, $\frac{2}{3}$, and $\frac{7}{11}$ to a common denominator.

4. Whence, if there be only two fractions proposed, multiply the terms of that having the less denominator by the quotient of their denominators.

EXAMPLES.

Reduce $\frac{2}{3}$ and $\frac{5}{11}$ to a common denominator.

$$\frac{2}{3} \text{ and } \frac{5}{11} = \frac{2 \times 4}{3 \times 4} \text{ and } \frac{5}{11} = \frac{8}{11} \text{ and } \frac{5}{11}.$$

Reduce $\frac{2}{3}$ and $\frac{5}{15}$ to a common denominator.

Reduce $\frac{4}{9}$ and $\frac{5}{17}$ to a common denominator.

VII. To reduce fractions to other equivalent ones of a different integer; a certain number of the less integer being contained in one of the greater.

RULE.

Consider how many of the less integer make one of the greater; and by that number, multiply the numerator if the reduction be to a less integer, or the denominator, if to a greater.

EXAMPLES.

Reduce $\frac{2}{9} l.$ to the fraction of a shilling.

$$\frac{2}{9} l. = \frac{2 \times 20}{9} s. = \frac{40}{9} s.$$

Reduce $\frac{40}{9} s.$ to the fraction of a $l.$

$$\frac{40}{9} s. = \frac{40}{9 \times 20} l. = \frac{2}{9} l.$$

Reduce $\frac{2}{11} l.$ to the fraction of a penny.

Reduce $32 d.$ or $\frac{1}{3} l.$ to the fraction of a $l.$

Reduce $\frac{5}{11} l.$ to the fraction of a farthing.

Reduce $\frac{1600}{7}$ of a farthing to the fraction of a $l.$

Reduce $\frac{2}{7} cwt.$ to the fraction of a $lb.$

Reduce $32 lb.$ or $\frac{1}{3} lb.$ to the fraction of a $cwt.$

Note, If a compound whole number be proposed, reduce it all to the lowest denomination mentioned in it, and proceed as before.

EX-

EXAMPLES.

Reduce 7s. 3d. to the fraction of a l. — *Ans.* $\frac{29}{80}$ l.

Reduce $2\frac{1}{4}$ d. to the fraction of a shilling. — *Ans.* $\frac{1}{48}$ s.

Reduce 3qr. 14lb. to the fraction of a cwt. — *Ans.* $\frac{7}{8}$ cwt.

VIII. To reduce fractions to equivalent ones of a different integer, when a certain number of the less are not exactly contained in the greater.

RULE.

1. By the last, reduce the given fraction to an equivalent one of such an integer, whereof a certain number are contained in the integer to which the fraction must be brought, or which shall contain a certain number of this.

2. By the last also, reduce this fraction to an equivalent one of the integer required.

EXAMPLES.

Reduce $\frac{2}{7}$ of a l. to the fraction of a guinea.

$$\frac{2}{7} l. = \frac{2 \times 20}{7} s. = \frac{2 \times 20}{7 \times 21} \text{gui.} = \frac{40}{147} \text{guineas.}$$

Reduce $\frac{1}{4}$ of a crown to the fraction of a guinea.

$$\text{Ans. } \frac{25}{108}$$

Reduce $\frac{40}{147}$ of a guinea to the fraction of a l. — *Ans.* $\frac{2}{7}$ l.

Reduce $\frac{1}{8}$ of a half-crown to the fraction of a shilling.

$$\text{Ans. } \frac{25}{112} \text{ or } 2\frac{1}{112} s.$$

Reduce $2\frac{1}{11} s.$ to the fraction of a half-crown.

$$\text{Ans. } \frac{5}{8} \text{ of a half-crown.}$$

IX. To find the value of proper fractions in numbers of inferior denominations.

RULE.

Multiply the numerator by the integer and divide by the denominator.

EXAMPLES.

1. What is the $\frac{4}{7}$ of 2l. 6s?

2l. 6s.

4

5) 9 4

Ans. — 1 l. 16s. 9d. $2\frac{2}{7} q.$

2. Re-

48 REDUCTION of VULGAR FRACTIONS.

2 Required the value of $\frac{2}{7}$ of a l.

$$\begin{array}{r} 2 \\ 20 \\ \hline \end{array}$$

3) 40 (13 s. 4 d. *Ans.*

$$\begin{array}{r} 1 \\ 12 \\ \hline \end{array}$$

12 (4 d.

3. Required the value of $\frac{1}{3}$ l. — *Ans.* 7 s. 6 d.
4. What is the value of $\frac{4}{11}$ l? — *Ans.* 6 s. 1 d. 3 $\frac{1}{11}$ q.
5. What is the value of $\frac{2}{3}$ of a guinea? — *Ans.* 4 s. 8 d.
6. What is the value of $\frac{1}{7}$ of a shilling? — *Ans.* 9 d. 1 $\frac{1}{7}$ q.
7. What is the value of $\frac{2}{7}$ of 9 s. 10 $\frac{1}{2}$ d? *Ans.* 1 s. 3 d. 3 $\frac{1}{7}$ q.
8. What is the value of $\frac{1}{4}$ of a lb. troy? — *Ans.* 9 oz.
9. What is the value of $\frac{1}{3}$ of a lb. avoirdupois? *Ans.* 12 oz.
10. What is the value of $\frac{5}{8}$ of a cwt? — *Ans.* 1 qr. 7 lb.
11. What is the value of $\frac{2}{7}$ of 3 cwt. 1 qr. 14 lb? *Ans.* 3 qr. 24 lb.
12. What is the value of $\frac{1}{17}$ of a mile? *Ans.* 1 furlong, 16 pls. 3 yd. 1 f. 9 $\frac{3}{17}$ in.
13. What is the value of $\frac{6}{7}$ of a yard? — *Ans.* 3 qr. 1 $\frac{1}{7}$ nl.
14. What is the value of $\frac{5}{8}$ of an acre? *Ans.* 1 rood, 2 $\frac{1}{8}$ pls.
15. What is the value of $\frac{1}{7}$ of a tun of wine? *Ans.* 3 hhd. 3 $\frac{1}{7}$ gal. 2 qrts.
16. What is the value of $\frac{2}{11}$ of a hhd. of ale? *Ans.* 6 gal. 3 $\frac{1}{11}$ qrts.
17. What is the value of $\frac{3}{8}$ of a quarter of corn? *Ans.* 4 bush. 1 pec. 1 gal. 2 $\frac{3}{8}$ qr.
18. What is the value of $\frac{1}{10}$ of a day? *Ans.* 7 hrs. 12 min.
19. What is the value of $\frac{1}{7}$ of a month? *Ans.* 2 we. 6 ds.
20. What is the value of $\frac{1}{10}$ of an ell english? *Ans.* 1 qr. 3 nls.

ADDITION of VULGAR FRACTIONS.

R U L E.

Reducer compound fractions to simple ones, and all to the same integer and denominator if they be different; then the sum of the numerators wrote over the common denominator will be the sum of the fractions required.

E X A M P L E S.

1. What is the sum of $\frac{5}{8}$, $7\frac{1}{2}$ and $\frac{1}{4}$ of $\frac{1}{2}$?
 $\frac{5}{8} + 7\frac{1}{2} + \frac{1}{4}$ of $\frac{1}{2} = \frac{5}{8} + 7\frac{1}{2} + \frac{1}{4} = \frac{5}{8} + 7\frac{4}{8} + \frac{2}{8} = 7\frac{5+4+2}{8} = 7\frac{11}{8} = 8\frac{3}{8}$ the sum.
2. What is the sum of $\frac{1}{2}$ and $\frac{4}{7}$? — *Ans.* $1\frac{9}{14}$.
3. What is the the sum of $\frac{2}{7}$ and $\frac{5}{14}$? — *Ans.* $\frac{9}{14}$.
4. What is the sum of $\frac{2}{7}$, $\frac{3}{7}$ and $\frac{4}{7}$? — *Ans.* $1\frac{9}{7}$.
5. What is the sum of $\frac{5}{9}$, $\frac{1}{3}$ and $2\frac{5}{9}$? — *Ans.* $3\frac{10}{9}$.
6. What is the sum of $\frac{3}{7}$, $\frac{4}{7}$ of $\frac{1}{3}$ and $9\frac{1}{18}$? *Ans.* $10\frac{29}{18}$.
7. What is the sum of $\frac{2}{3}$ of a pound, and $\frac{1}{6}$ of a shilling? — — *Ans.* $\frac{125}{2}$ s. or 13 s. 10 d. $2\frac{2}{3}$ q.
8. What is the sum of $\frac{1}{3}$ s. and $\frac{4}{11}$ d? *Ans.* $\frac{113}{33}$ d. or 7 d. $1\frac{1}{33}$ q.
9. What is the sum of $\frac{1}{7}$ l. $\frac{2}{9}$ s. and $\frac{5}{12}$ d? *Ans.* $\frac{3139}{1008}$ s. or 3 s. 1 d. $1\frac{10}{112}$ q.
10. Suppose that I have $\frac{3}{8}$ of a ship worth 1500 l. and that I buy another person's share of her, which is $\frac{5}{8}$; what part of her belongs to me then, and what is it worth? — — *Ans.* I have $\frac{11}{8} = 1031$ l. 5 s.

SUBTRACTION of VULGAR FRACTIONS.

RULE.

THE same preparations being made here as in addition; the difference of the numerators wrote above the common denominator will be the difference of the fractions required.

Note, in subtracting mixt numbers, when the fraction in the subtrahend is greater than that in the minuend, subtract the numerator of the subtrahend from the denominator, and to the difference add the numerator of the minuend; and carry one to the integer in the subtrahend.

EXAMPLES.

1. What is the difference between $\frac{5}{8}$ and $\frac{1}{8}$?

$$\frac{5}{8} - \frac{1}{8} = \frac{5-1}{8} = \frac{4}{8} = \frac{1}{2} \text{ Ans.}$$

2. What is the difference between $\frac{15}{17}$ and $\frac{11}{17}$?

$$\frac{15}{17} - \frac{11}{17} = \frac{15-11}{17} = \frac{4}{17} \text{ Ans.}$$

3. What is the diff. between $\frac{3}{12}$ and $\frac{7}{12}$? — *Ans.* $\frac{4}{12}$.

4. What is the diff. between $\frac{1}{11}$ and $\frac{4}{11}$? — *Ans.* $\frac{3}{11}$.

5. What is the diff. between $\frac{3}{12}$ and $\frac{7}{12}$? — *Ans.* $\frac{4}{12}$.

6. What is the diff. between $5\frac{1}{4}$ and $\frac{3}{4}$ of $4\frac{1}{2}$?

$$\text{Ans. } 4\frac{3}{4}.$$

7. What is the diff. between $\frac{3}{4}$ of a *l.* and $\frac{1}{4}$ of $\frac{1}{4}$ of a shilling? — *Ans.* $\frac{1}{4}$ s. or 10s. 7d. 1 $\frac{1}{4}$ q.

8. What is the diff. between $\frac{3}{4}$ of $5\frac{1}{2}$ *l.* and $\frac{1}{4}$ of a shilling? — — *Ans.* $\frac{1}{4}$ s. or 1l. 8s. 11 $\frac{1}{4}$ d.

9. Suppose that I have $\frac{1}{4}$ of a ship which is worth 900*l.* and that I sell $\frac{3}{4}$ of my share; what part of her have I left, and what is it worth? *Ans.* $\frac{1}{4} = 187\frac{1}{2}$ *l.* 10s.

MUL.

MULTIPLICATION of VULGAR FRACTIONS.

R U L E.

Reducè mixt numbers, if there be any, to fractions; then multiply all the numerators together for the numerator, and all the denominators together for the denominator of the product required.

Note, A fraction is best multiplied by an integer, by dividing the denominator by it if possible; but if not, multiply the numerator by it.

E X A M P L E S.

1. What is the product of $\frac{2}{3}$, $3\frac{1}{2}$, 5, and $\frac{3}{4}$ of $\frac{1}{2}$?
 $\frac{2}{3} \times 3\frac{1}{2} \times 5 \times \frac{3}{4}$ of $\frac{1}{2} = \frac{2 \times 7 \times 5 \times 3 \times 1}{3 \times 4 \times 4 \times 2} = \frac{1 \times 3 \times 5}{2 \times 4} = \frac{15}{8} = 4\frac{7}{8}$ *Ans.*
2. What is the product of $\frac{2}{7}$ and $\frac{5}{8}$? — *Ans.* $\frac{5}{28}$.
3. What is the product of $\frac{4}{11}$ and $\frac{5}{4}$? — *Ans.* $\frac{5}{11}$.
4. What is the product of $\frac{1}{7}$, $\frac{4}{9}$ and $\frac{14}{11}$? — *Ans.* $\frac{4}{11}$.
5. What is the product of $\frac{1}{2}$, $\frac{2}{3}$ and 3? — *Ans.* 1.
6. What is the product of $\frac{1}{14}$ and 7? — *Ans.* $1\frac{1}{2}$.
7. What is the product of $\frac{7}{9}$, $\frac{1}{3}$ and $4\frac{5}{14}$? — *Ans.* $2\frac{1}{18}$.
8. What is the product of $\frac{5}{8}$, and $\frac{2}{3}$ of $\frac{6}{7}$? — *Ans.* $\frac{10}{49}$.
9. What is the product of $5\frac{1}{11}$, and 9? — *Ans.* 48.
10. What is the product of 6, and $\frac{2}{3}$ of 5? — *Ans.* 20.
11. What is the product of $\frac{2}{9}$ of $\frac{1}{7}$, and $\frac{5}{8}$ of $3\frac{2}{7}$?
Ans. $\frac{5}{14}$.
12. What is the product of $3\frac{2}{7}$, and $4\frac{1}{11}$?
Ans. $14\frac{24}{11}$.
13. What is the product of 5, $\frac{2}{11}$, $\frac{2}{7}$ of $\frac{1}{3}$, and $4\frac{1}{8}$?
Ans. $2\frac{9}{11}$.

DIVISION of VULGAR FRACTIONS.

R U L E.

Having prepared the terms as in multiplication; take the quotient of the numerators and of the de-

52 RULE-OF-THREE in VULGAR FRACTIONS.

nominators, if possible, for the numerator and denominator of the fraction required; but if that cannot be done, multiply the dividend by the *reciprocal* of the divisor, for the quotient required.

Note 1. By the reciprocal of a fraction, is meant the fraction got by inverting its terms: so the reciprocal of $\frac{2}{3}$ is $\frac{3}{2}$ and of $\frac{5}{7}$ or $\frac{1}{4}$ is $\frac{4}{5}$.

2. A fraction is divided by an integer by dividing the numerator by it, if possible; but if not, multiply the denominator by it.

EXAMPLES.

1. What is the quotient of $\frac{25}{9}$ by $\frac{5}{3}$?

$$\frac{25}{9} \div \frac{5}{3} = \frac{25 \div 5}{9 \div 3} = \frac{5}{3} = 1\frac{2}{3} \text{ Ans.}$$

2. What is the quotient of $\frac{5}{9}$ by $\frac{2}{15}$?

$$\frac{5}{9} \div \frac{2}{15} = \frac{5}{9} \times \frac{15}{2} = \frac{5 \times 15}{9 \times 2} = \frac{3 \times 5 \times 3}{3 \times 2} = \frac{5 \times 3}{2} = \frac{15}{2} = 7\frac{1}{2} \text{ Ans.}$$

3. What is the quotient of $\frac{15}{8}$ by $\frac{4}{7}$? — *Ans.* $\frac{4}{3}$.

4. What is the quotient of $\frac{7}{8}$ by $\frac{1}{4}$? — *Ans.* $\frac{7}{2}$.

5. What is the quotient of $\frac{14}{9}$ by $\frac{7}{8}$? — *Ans.* $1\frac{1}{3}$.

6. What is the quotient of $\frac{5}{8}$ by $\frac{1}{7}$? — *Ans.* $\frac{35}{8}$.

7. What is the quotient of $\frac{11}{12}$ by $\frac{3}{4}$? — *Ans.* $\frac{11}{9}$.

8. What is the quotient of $\frac{3}{7}$ by $\frac{1}{3}$? — *Ans.* $\frac{9}{7}$.

9. What is the quotient of $\frac{9}{8}$ by 3? — *Ans.* $\frac{3}{8}$.

10. What is the quotient of $\frac{3}{7}$ by 7? — *Ans.* $\frac{3}{49}$.

11. What is the quotient of 5 by $\frac{7}{10}$? — *Ans.* $7\frac{1}{7}$.

12. What is the quotient of $7\frac{2}{3}$ by $9\frac{5}{9}$? — *Ans.* $\frac{34}{45}$.

13. What is the quotient of $\frac{2}{3}$ of $\frac{3}{9}$ by $\frac{5}{7}$ of $7\frac{1}{2}$?

$$\text{Ans. } \frac{7}{111}.$$

RULE-OF-THREE in VULGAR FRACTIONS.

RULE.

HAVING made the necessary preparations for multiplication, multiply continually together the 2d and 3d terms and the reciprocal of the 1st, for the answer.

EX.

EXAMPLES.

1. If $\frac{1}{4}$ of a yard of velvet cost $\frac{1}{2}$ l. what will $\frac{1}{8}$ of a yard cost?

$$\begin{array}{ccccccc} \text{yd.} & & \text{l.} & & \text{yd.} & & \text{l.} \\ \frac{1}{8} & \text{---} & \frac{1}{2} & \text{---} & \frac{1}{8} & \text{---} & \frac{1}{2} \\ & & & & \frac{1 \times 1 \times 8}{16 \times 5 \times 1} & = & \frac{1}{5} = 6s. 8d. \text{ Ans.} \end{array}$$

2. What will $3\frac{1}{4}$ oz. of silver cost at 6s. 4d. an ounce?
Ans. 1l. 1s. 4 $\frac{1}{2}$ d.

3. If $\frac{3}{8}$ of a ship be worth 273l. 2s. 6d. what is $\frac{1}{16}$ of her worth? — — Ans. 227l. 12s. 1d.

4. What will $13\frac{3}{4}$ lb. cost at the rate of 17 $\frac{1}{4}$ l. per cwt?
Ans. 2l. 3s. 3 $\frac{1}{4}$ d.

5. What is the purchase of 1230l. bank-stock, at 108 $\frac{1}{4}$ per cent? — — — Ans. 1336l. 1s. 9d.

6. What is the interest of 273l. 15s. for a year, at 3 $\frac{1}{4}$ per cent? — — — Ans. 8l. 17s. 11 $\frac{1}{4}$ d.

7. If $\frac{1}{4}$ of a ship be worth 73l. 1s. 3d. what part of her may I buy for 250l. 10s? — — — Ans. $\frac{3}{4}$ of her.

8. What must be paid for 5 $\frac{1}{2}$ oz. at the rate of 5 $\frac{1}{2}$ s. per lb. troy? — — — Ans. 2s. 6 $\frac{1}{8}$ d.

9. How much india-stock may be bought for 3041l. 2s. 3d. at 172 $\frac{1}{4}$ l. per cent? — — — Ans. 1760l. 8s. 2d. 3 $\frac{1}{8}$ q.

10. What does the commission of 530l. 2s. 9d. amount to at 2s. 6d. per cent? — — — Ans. 13s. 3 $\frac{1}{8}$ d.

11. How much flemish money must be given for 273l. 6s. 8d. sterling, at the rate of 34 $\frac{1}{2}$ 6d. flemish per pound sterling? — — — Ans. 471l. 10s.

12. How much south-sea stock at 111 $\frac{1}{4}$ per cent. will 10000l. purchase? — — — Ans. 8978l. 13s. 6 $\frac{1}{8}$ d.

13. How much sterling money must be given for 471l. 10s. flemish, at the rate of 34 $\frac{1}{2}$ 6d. flemish for each pound sterling? — — — Ans. 273l. 6s. 8d.

RULE - OF - FIVE in VULGAR FRACTIONS.

RULE.

TAKE the continual product of the three last and reciprocals of the two first terms, for the answer required.

EXAMPLES.

1. If 2*l.* 10*s.* be the wages of 15 men for 6 days, what will be the wages of 12 men for $18\frac{1}{7}$ days?

$$\left. \begin{array}{l} 15 \text{ men} \\ 6 \text{ days} \end{array} \right\} - \frac{5}{1} \text{ l.} - \left\{ \begin{array}{l} 12 \text{ men} \\ 5\frac{1}{7} \text{ days} \end{array} \right\} - 12 \times \frac{5}{1} \times \frac{5}{1} \times \frac{1}{17} \times \frac{1}{8} = \frac{12 \times 5 \times 5 \times 1 \times 1}{1 \times 1 \times 17 \times 8} = \frac{55}{136} = \frac{55}{8} \text{ l.} = 6 \text{ l. } 2 \text{ s. } 2 \text{ d. } 2\frac{1}{2} \text{ q. } \text{Ans.}$$

2. What is the interest of 35*q.* for 18 months, at 5 per cent. per annum? ———— *Ans.* 2*l.* 5*s.*

3. If I pay 16*s.* 4*d.* for the carriage of $5\frac{1}{4}$ *c. wt.* 20 miles, what must be paid for the carriage of $17\frac{1}{2}$ *c. wt.* $7\frac{1}{2}$ miles? ———— *Ans.* 1*l.* 8*s.* 4*d.*

4. If a footman travel 273 miles in $6\frac{1}{7}$ days of 12 hours long, in how many days of $9\frac{1}{4}$ hours each may he travel 132 miles? ———— *Ans.* $2\frac{1}{3}\frac{7}{11}$ days.

DECIMAL FRACTIONS.

A Decimal is a fraction whose denominator is 1 with some number of ciphers annexed; as $\frac{1}{10}$, or $\frac{45}{10000}$.

Decimals are wrote down without their denominators, the numerators being so distinguished as to evince what the denominators are; which is done by separating, by a point, so many of the right-hand figures from the rest as there are ciphers in the denominator; the figures on the left-hand of the point being integers, and those on the right, decimals: So $\frac{13}{10}$ is wrote 1.3, and named 1 and 3 tenths; $\frac{15769}{10000}$ is wrote 15.769, and named 15 and 769 thousandth-parts; and $\frac{25}{10000}$ is wrote .25, and named 25 hun-

ADDITION and SUBTRACTION of DECIMALS. 55

hundredths or *hundredth-parts*. — But if there be not a sufficient number of figures in the numerator, cyphers are prefixed to supply the defect: So $\frac{1}{100}$ is wrote $\cdot 01$, that is 1 *hundredth*; and $\frac{15}{10000}$ thus $\cdot 0015$, that is 15 *ten-thousandths*.

NOTE 1. The 1st, 2d, 3d, 4th, &c. places of decimals, counting from the left-hand towards the right, are denominated the places of *primes*, *seconds*, *thirds*, and *fourths*, &c. respectively.

2. Cyphers on the right of decimals do not affect their value.

ADDITION and SUBTRACTION of DECIMALS.

WRITE the proposed numbers under each other, according to the value of their places, as in whole numbers; in which order the decimal points will stand directly below each other: then add or subtract as in whole numbers, putting a decimal point in the sum or difference strait below the other points.

EXAMPLES in Addition.

1. What is the sum of 276, 39 \cdot 213, 72014 \cdot 9, 417, and 5032?
2. What is the sum of 7530, 16 \cdot 201, 3 \cdot 0142, 957 \cdot 13, 6 \cdot 72819, and \cdot 03014?
3. What is the sum of 312 \cdot 09, 3 \cdot 5711, 4195 \cdot 6, 71 \cdot 498, 9739 \cdot 215, 179 and \cdot 0027?
4. What is the sum of \cdot 014, \cdot 9816, \cdot 32, \cdot 15914, 72913, and \cdot 0047?
5. What is the sum of 27 \cdot 148, 918 \cdot 73, \cdot 14016, 295304, \cdot 713826, and 291 \cdot 7?

EXAMPLES in Subtraction.

1. What is the difference between \cdot 9173 and \cdot 2138?
2. What is the difference of 1 \cdot 9185 and 2 \cdot 73?
3. What is the difference of 214 \cdot 81 and 4 \cdot 90142?
4. What is the difference of 91 \cdot 713 and 407?
5. What is the difference of 2714 and \cdot 916?

MUL-

MULTIPLICATION of DECIMALS.

WRITE down the factors and multiply exactly as in integers, placing the decimal point in the product so as to make just as many decimals in it as there are in both factors ; and if there be not so many figures in the product as there ought to be decimals, prefix cyphers to supply the defect.

E X A M P L E S.

1. What is the product of $\cdot 417$ and $520\cdot 3$?
2. What is the product of $91\cdot 78$ and $\cdot 381$?
3. What is the product of $\cdot 217$ and $\cdot 053$?
4. What is the product of $51\cdot 6$ and 21 ?
5. What is the product of 314 and $\cdot 029$?
6. What is the product of $\cdot 051$ and 009 ?

C O N T R A C T I O N S.

(1.) *When decimals are to be multiplied by 1 with any number of cyphers ; it is done by only removing the decimal point so many places farther to the right hand as there are cyphers in the multiplier, and subjoining cyphers if need be.*

E X A M P L E S.

1. The product of $51\cdot 3$ and 1000 is 51300 .
2. The product of $2\cdot 714$ and 100 is
3. The product of $\cdot 9163$ and 1000 is
4. The product of $21\cdot 31$ and 10000 is

(2.) *When the product will contain many more decimals than are necessary for the present purpose, the work may be contracted thus :*

Write the units figure of the multiplier streight under such decimal place of the multiplicand as you intend the last of your product shall be, writing the other figures of the multiplier in an inverted order ; then in multiplying reject all the figures in the multiplicand which are on the right of the figure you are multiplying by ; writing the products down so, that their right hand figures fall streight below each other ; and carrying to such right hand figures

gures from the product of the two preceding figures in the multiplicand thus: viz. 1 from 5 to 15, 2 from 15 to 25, 3 from 25 to 35, &c. and the sum of the lines will be the product to the number of decimals required, and will be seldom wrong in the last figure.

EXAMPLES.

1. Multiply 27.14986 by 92.41035, so as to retain only four places of decimals in the product.

Contracted.

$$\begin{array}{r}
 27.14986 \\
 53014.29 \\
 \hline
 24434874 \\
 542997 \\
 108599 \\
 2715 \\
 81 \\
 14 \\
 \hline
 2508.9280
 \end{array}$$

Common way.

$$\begin{array}{r}
 27.14986 \\
 92.41035 \\
 \hline
 13 \quad 574930 \\
 81 \quad 44958 \\
 2714 \quad 986 \\
 108599 \quad 44 \\
 542997 \quad 2 \\
 24434874 \\
 \hline
 2508.9280 \quad | \quad 650510
 \end{array}$$

2. Multiply 480.14936 by 2.72416, retaining four decimals in the product.

3. Multiply 2490.3048 by .573286, retaining five decimals in the product.

4. Multiply 325.701428 by .7218393, retaining three decimals in the product.

DIVISION of DECIMALS.

Divide as in integers; and to know how many decimals must be in the quotient, observe the following rules.

RULE 1.

The first figure of the quotient must possess the same place, of decimals or integers, as doth that figure of the dividend under which the units place of the first figure's product stands.

RULE 2.

The decimal places of the divisor and quotient together

ther must be equal in number to those in the dividend:—
Whence

1. *If the number of decimal places in the divisor and dividend be equal, the quotient will be integral.*

Note, If, in this, or the following cases, there be a remainder after all the dividend figures are used, the quotient may be continued to what number of decimals you please by adjoining a cipher continually to the last remainder.

EXAMPLES.

$$\begin{array}{r} \cdot 14) 720193 (\\ 7 \cdot 13) \cdot 18 (\end{array} \quad \begin{array}{r} 3 \cdot 75) 3 \cdot 15 (\\ \cdot 285) \cdot 109 (\end{array}$$

2. *If the number of decimals in the dividend exceed that of those in the divisor, point off in the quotient so many as their difference is; and if there be not so many places in the quotient, prefix so many ciphers as will make up the number.*

EXAMPLES.

$$\begin{array}{r} 8 \cdot 4) 3 \cdot 15 (\\ 519) \cdot 3049 (\end{array} \quad \begin{array}{r} 217) 380 \cdot 2196 (\\ \cdot 216) \cdot 20913 (\end{array}$$

3. *When the number of decimals in the dividend is less than that in the divisor, annex ciphers to those in the dividend till they be equal; then will the quotient be integral, as in case 1.*

EXAMPLES.

$$\begin{array}{r} \cdot 216) 7142 (\\ \cdot 00401) 2168 (\end{array} \quad \begin{array}{r} 4 \cdot 913) 758 \cdot 6 (\\ 8 \cdot 73) 140896 (\end{array}$$

CONTRACTIONS.

(1.) *If the divisor be an integer with any number of ciphers at the end, cut them off, and remove the decimal point in the dividend so many places farther to the left as there were ciphers cut off, prefixing ciphers if need be; then proceed as before.*

EXAMPLES.

$$\begin{array}{r} 2170) 45 \cdot 5 (\\ 21000) 953 (\end{array} \quad \begin{array}{r} 32000) 41020 (\\ 79000) 61 (\end{array}$$

(2.) *Whence, if the divisor be 1 with ciphers, the quotient will be the same figures with the dividend, having the decimal point so many places farther to the left as there are ciphers in the divisor.*

E x -

EXAMPLES.

$$217.3 \div 100 = 2.173$$

$$419 \text{ by } 10 =$$

$$5.16 \text{ by } 1000 =$$

$$.21 \text{ by } 1000 =$$

(3.) *When the number of figures in the divisor is great, the division at large will be very troublesome, but may be contracted thus:*

Having by the first general rule found what place of decimals or integers the first figure of the quotient will possess; consider how many figures of the quotient will serve the present purpose, then take the same number of the left-hand figures of the divisor, and so many of the dividend figures as will contain them (less than 10 times); by these find the first figure of the quotient, and for each following figure divide the last remainder by the divisor wanting one figure to the right more than before, but observing what must be carried to the first product for such omitted figures as in the second contraction of multiplication; and continue the operation till the divisor be exhausted.

Note, When there are not so many figures in the divisor as are required to be in the quotient, begin the division with all the figures as usual, and continue it till the number of figures in the divisor and of those remaining to be found in the quotient be equal, after which use the contraction.

EXAMPLES.

1. Divide 2508.928065051 by 92.41035 so as to have four decimals in the quotient, in which case the quotient will contain six figures.

$$92.4103,5) 2508.928,065051 \text{ (27.1498.)}$$

$$\begin{array}{r} 660721 \\ \hline \end{array}$$

$$\begin{array}{r} 13849 \\ \hline \end{array}$$

$$\begin{array}{r} 4608 \\ \hline \end{array}$$

$$\begin{array}{r} 912 \\ \hline \end{array}$$

$$\begin{array}{r} 80 \\ \hline \end{array}$$

$$\begin{array}{r} 6 \\ \hline \end{array}$$

2. Divide 4109.2351 by 230.409 so that the quotient may contain four decimals.

3. Divide 37.10438 by 5713.96 that the quotient may contain five decimals.

4. Divide 913.08 by 2137.2 that the quotient may contain three decimals.

REDUCTION of DECIMALS.

* I. **T**O reduce a vulgar fraction to an equivalent decimal.

R U L E.

Divide the numerator by the denominator as in division of decimals, and the quotient will be the decimal required.

E X A M P L E S.

Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$, $\frac{1}{7}$, $\frac{1}{8}$, and $\frac{1}{9}$ to decimals.

Re-

* Since to throw any vulgar fraction, whose denominator is a prime number greater than what it is common to divide by in one line, into a decimal consisting of a great number of figures, has engaged the attention of many eminent persons, I shall here put down the method which Mr Colson has given in pag. 162 of Sir Isaac Newton's fluxions; which method performs the work much sooner than any other that I know of.

The method will be best explained by an example, thus: "Suppose (for Instance) I would find the reciprocal of the prime number 29, or the value of the fraction $\frac{1}{29}$ in decimal numbers. I divide 1.00000 &c. by 29, in the common way, so far as to find two or three of the first figures, or till the remainder becomes a single figure, and then I assume the supplement to complete the quotient. Thus I shall have $\frac{1}{29} = 0.03448\frac{8}{29}$ for the complete quotient, which equation if I multiply by the numerator 8, it will give $\frac{8}{29} = 0.27584\frac{4}{29}$, or rather $\frac{8}{29} = 0.27586\frac{6}{29}$. I substitute this instead of the fraction in the first equation, and I shall have $\frac{1}{29} = 0.0344827586\frac{6}{29}$. Again, I multiply this equation by 6, and it will give $\frac{6}{29} = 0.2068965517\frac{7}{29}$, and then by substitution $\frac{1}{29} = 0.03448275862068965517\frac{7}{29}$. Again, I multiply this equation by 7, and it becomes $\frac{7}{29} = 0.24137931034482758620\frac{2}{29}$, and then by substitution $\frac{1}{29} = 0.0344827586206896551724137931034482758620\frac{2}{29}$, where every operation will at least double the number of figures found by the preceding operation, And this will be an easy expedient for converting division into multiplication in all cases. For this reciprocal of the divisor being thus found, it may be multiplied into the dividend to produce the quotient."

Reduce $\frac{3}{7}$ to a decimal.

Reduce $\frac{1}{100}$ to a decimal.

Reduce $\frac{3}{738}$ to a decimal.

II. To reduce integers or decimals to equivalent decimals of superior denominations.

CASE I.

If a simple number or decimal be proposed, reduce it to the name required by dividing as in reduction of integers.

EXAMPLES.

1. Reduce 1 *dwt.* to the decimal of a *lb.*
Ans. .004166 &c. *lb.*
2. Reduce 9*d.* to the decimal of a pound.
Ans. .0375*l.*
3. Reduce 7 drams to the decimal of a *lb.* avoird.
Ans. .02734375*lb.*
4. Reduce .26*d.* to the decimal of a *l.*
Ans. .0010833 &c. *l.*
5. Reduce 2.15 *lb.* to the decimal of a *cwt.*
Ans. .019196 + *cwt.*
6. Reduce 24 yards to the decimal of a mile.
Ans. .013636 &c. *mls.*
7. Reduce .056 poles to the decimal of an acre.
Ans. .00035 *acr.*
8. Reduce 1.2 pints of wine to the decimal of a *hhd.*
Ans. .00238 + *hhd.*
9. Reduce 14 minutes to the decimal of a day.
Ans. .009722 &c. *da.*
10. Reduce .21 pints to the decimal of a peck.
Ans. .013125 *pec.*

CASE 2.

A compound number may be reduced to a superior name by reducing each of its parts, and taking the sum of the decimals; the best way to do which is thus:

Write the given numbers under each other, proceeding orderly from the least to the greatest name, for dividends; draw a perpendicular line on the left of these, and on the left of it write opposite each dividend such a number, for

G

a di-

a divisor, as will reduce it to the next superior name; then begin with the upper division, and affix the quotient of each to the next dividend, as a decimal part of it, before it be divided, and the last sum will be the answer.

E X A M P L E S.

1. Reduce 3*l.* 12*s.* 6½*d.* to the denomination of pounds.

$$\begin{array}{r|l} 4 & 3 \\ 12 & 6.75 \\ 20 & 12.5625 \\ & 3.628125 \text{ Ans.} \end{array}$$

2. Reduce 19*l.* 17*s.* 3¼*d.* to *l.*

Ans. 19.86354166 &c. *l.*

3. Reduce 15*s.* 6*d.* to the decimal of *l.*—*Ans.* .775*l.*

4. Reduce 7½*d.* to the decimal of a shill.—*Ans.* .625*s.*

5. Reduce 5*oz.* 12*dwt.* 16*gr.* to *lbs.*

Ans. .46944 &c. *lb.*

6. Reduce 3*cwt.* 2*qr.* 14*lb.* to *cwt.* *Ans.* 3.625*cwt.*

7. Reduce 17*yd.* 1*ft.* 6*in.* to the decimal of a mile.

Ans. .0099431818 &c. *mil.*

8. Reduce 2*qr.* 3*nls.* to the decimal of a yard.

Ans. .6875*yd.*

9. Reduce 13*acr.* 1*ro.* 14*pol.* to acres.

Ans. 13.3375*acr.*

10. Reduce 13*gal.* 1*pint* of wine to the decimal of a *hhd.* ——— *Ans.* .20833 &c. *hhd.*

11. Reduce 3*busb.* 1*pec.* to the decimal of a quarter.

Ans. .40625*qr.*

12. Reduce 3*mon.* 1*we.* 5*da.* to months.

Ans. 3.42857+*mon.*

III. To reduce a decimal of a superior denomination to its value in the inferior ones.

R U L E.

Multiply the given decimal by such a number as will reduce it to the next inferior name, and point off in the product so many places of decimals as are in the given number; then reduce these in the same manner to the next

next name, and continue the reduction to the lowest name required, or till the decimals pointed off be all ciphers: then the numbers on the left of the points will express the value of the decimal.

EXAMPLES.

1. What is the value of $\cdot 775$ l? — *Ans.* 15s. 6d.
2. What is the value of $\cdot 625$ shil? — *Ans.* $7\frac{1}{2}$ d.
3. What is the value of $\cdot 8635$ l. — *Ans.* 17s. 3 \cdot 24d.
4. What is the value of $\cdot 0125$ lb. troy? — *Ans.* 3 dwts.
5. What is the value of $\cdot 4694$ lb. troy?
Ans. 5 oz. 12 dwts. 15 \cdot 744 gr.
6. What is the value of $\cdot 625$ cwt? — *Ans.* 2 qr. 14 lb.
7. What is the value of $\cdot 009943$ mil.
Ans. 17 yd. 1 ft. 5 \cdot 98848 inc.
8. What is the value of $\cdot 6875$ yd? — *Ans.* 2 qr. 3 nls.
9. What is the value of $\cdot 3375$ acr?
Ans. 1 rd. 14 poles.
10. What is the value of $\cdot 2083$ hhd. of wine?
Ans. 13 \cdot 1229 gal.
11. What is the value of $\cdot 49625$ qr. of corn?
Ans. 3 busb. 1 pack.
12. What is the value of $\cdot 42857$ mon?
Ans. 1 we. 4 ds. 23 \cdot 99904 hrs.

RULE-OF-THREE in DECIMALS.

REDUCE vulgar fractions to decimals, and compound numbers either to decimals of the higher names or to integers of the lower, as also the first and third to the same name: then state the question and proceed as in integers.

Note. Any of the convenient examples of the rules of three or five in integers or vulgar fractions may be taken as proper examples of the same rules in decimals; for it would be filling the book to ill purpose to give different examples here.—The following example, which is the first in vulgar fractions, is wrought here to shew the method.

If $\frac{3}{4}$ of a yard of velvet cost $\frac{2}{5}l.$ what will $\frac{5}{8}yd.$ cost?

	<i>yd.</i>	<i>l.</i>	<i>yd.</i>
$\frac{3}{4} = .375$.375	— .4 —	.3125
			.4
$\frac{2}{5} = .4$.375) 12500 (333333 &c.
			1125 20
$\frac{5}{8} = .3125$			1250 s. 6.66666 &c.
			1125 12
			125 d. 7.99999 &c.

Note, The remainder, in the division, being always the same, the quotient figure must be so likewise; so that if the quotient were infinitely continued it would be equal to $\frac{1}{4}l.$ as in vulgar fractions.

RULE-OF-FIVE *in* DECIMALS.

THE same preparations must be made here as in the rule-of-three before the stating.

PRACTICE.

BY *rules of practice* are meant certain expeditious methods of casting up accounts: and they consist of the most general contractions of the rule-of-three when the first term is 1.

Note 1. One number is said to be an aliquot part of another, when the former divides the latter without a remainder.

2. When the quantity, whose price is to be found, is not very large, and of one denomination, it is commonly best to work it by compound multiplication; but if very large, or of several denominations, use the following rules of practice.

A TABLE of the aliquot parts of money.

s.	d.	l.	d.	l.	s.
10	—	is	$\frac{1}{2}$	6	is $\frac{1}{40}$ or $\frac{1}{2}$
6	8	—	$\frac{1}{3}$	5	— $\frac{1}{24}$
5	—	—	$\frac{1}{4}$	4	— $\frac{1}{30}$ — $\frac{1}{3}$
4	—	—	$\frac{1}{5}$	$3\frac{1}{2}$	— $\frac{1}{24}$
3	4	—	$\frac{1}{6}$	3	— $\frac{1}{30}$ — $\frac{1}{4}$
2	6	—	$\frac{1}{8}$	$2\frac{1}{2}$	— $\frac{1}{24}$
2	—	—	$\frac{1}{10}$	2	— $\frac{1}{30}$ — $\frac{1}{6}$
1	8	—	$\frac{1}{12}$	$1\frac{1}{2}$	— $\frac{1}{30}$ — $\frac{1}{8}$
1	4	—	$\frac{1}{15}$	$1\frac{1}{4}$	— $\frac{1}{24}$
1	3	—	$\frac{1}{18}$	1	— $\frac{1}{40}$ — $\frac{1}{12}$
1	—	—	$\frac{1}{20}$	$\frac{3}{4}$	— $\frac{1}{30}$ — $\frac{1}{6}$
10	—	—	$\frac{1}{24}$	$\frac{1}{2}$	— $\frac{1}{40}$ — $\frac{1}{4}$
8	—	—	$\frac{1}{30}$	$\frac{1}{4}$	— $\frac{1}{60}$ — $\frac{1}{3}$
$7\frac{1}{2}$	—	—	$\frac{1}{12}$		

RULE I.

If the given price of 1 or the integer be an aliquot part of a penny, shilling, or pound, take the same part of the given quantity whose price is to be found (by dividing it by the number of times which the given price of 1 is contained in a penny, shilling, or pound) for the answer in pence, shillings, or pounds respectively.

EXAMPLES.

Questions.			Answers.		
s.	d.		l.	s.	d.
2103	at 10	—	1051	10	—
715	at 6 8	—	238	6	8
1496	at 5	—	374	—	—
420	at 4	—	84	—	—
831	at 3 4	—	138	10	—
275	at 2 6	—	34	7	6
937	at 2	—	93	14	—
G 3			3890		

<i>Questions.</i>				<i>Answers.</i>			
	<i>s.</i>	<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	
3890 at	1	8	— —	324	3	4	
624 at	1	4	— —	41	12	—	
159 at	1	3	— —	9	18	9	
472 at	1	—	— —	23	12	—	
521 at	—	10	— —	21	14	2	
376 at	—	8	— —	12	10	8	
916 at	—	$7\frac{1}{2}$	— —	28	12	6	
1270 at	—	6	— —	31	15	—	
835 at	—	5	— —	17	7	11	
609 at	—	4	— —	10	3	—	
581 at	—	$3\frac{1}{4}$	— —	9	1	$6\frac{1}{4}$	
716 at	—	3	— —	8	19	—	
1953 at	—	$2\frac{1}{2}$	— —	20	6	$10\frac{1}{2}$	
807 at	—	2	— —	6	14	6	
2305 at	—	$1\frac{1}{2}$	— —	14	8	$1\frac{1}{2}$	
3019 at	—	$1\frac{1}{4}$	— —	15	14	$5\frac{1}{4}$	
4718 at	—	1	— —	19	13	2	
1927 at	—	$\frac{3}{4}$	— —	6	—	$5\frac{1}{4}$	
741 at	—	$\frac{1}{2}$	— —	1	10	$10\frac{1}{2}$	
5804 at	—	$\frac{1}{4}$	— —	6	—	11	

RULE II.

If the given price be no aliquot part of a penny, shilling, or pound; divide it into several aliquot parts, then work for each by rule 1, and their sum will be the answer.

Or, It may often be divided so, that the less will be aliquot parts of the greater; then take the same parts of the prices found for the greater.

EXAMPLES.

<i>Questions.</i>				<i>Answers.</i>			
		<i>d.</i>		<i>l.</i>	<i>s.</i>	<i>d.</i>	
2173	— at	$1\frac{1}{2}$	— —	15	16	$10\frac{1}{4}$	
796	— at	$2\frac{1}{4}$	— —	7	9	3	
				1953			

Questions.				Answers.		
	s.	d.		l.	s.	d.
1953 at —	—	$2\frac{1}{4}$	—	22	7	$6\frac{1}{4}$
3406 at —	—	$3\frac{1}{4}$	—	46	2	$5\frac{1}{4}$
596 at —	—	$3\frac{1}{2}$	—	8	13	10
802 at —	—	$4\frac{1}{4}$	—	14	4	$—\frac{1}{2}$
1234 at —	—	$4\frac{1}{2}$	—	23	2	9
370 at —	—	$4\frac{3}{4}$	—	7	6	$5\frac{1}{2}$
951 at —	—	$5\frac{1}{4}$	—	20	16	$—\frac{1}{4}$
603 at —	—	$5\frac{1}{2}$	—	13	6	$4\frac{1}{2}$
239 at —	—	$5\frac{3}{4}$	—	5	14	$6\frac{1}{4}$
1980 at —	—	$6\frac{1}{4}$	—	51	11	3
271 at —	—	$6\frac{1}{2}$	—	7	6	$9\frac{1}{2}$
714 at —	—	$6\frac{3}{4}$	—	20	1	$7\frac{1}{4}$
591 at —	—	7	—	17	4	9
275 at —	—	$7\frac{1}{4}$	—	8	6	$1\frac{1}{4}$
596 at —	—	$7\frac{3}{4}$	—	19	4	11
1490 at —	—	$8\frac{1}{4}$	—	51	4	$4\frac{1}{2}$
621 at —	—	$8\frac{1}{2}$	—	21	19	$10\frac{1}{2}$
4681 at —	—	$8\frac{3}{4}$	—	170	13	$2\frac{3}{4}$
753 at —	—	9	—	28	4	9
421 at —	—	$9\frac{1}{4}$	—	16	4	$6\frac{1}{4}$
210 at —	—	$9\frac{1}{2}$	—	8	6	3
765 at —	—	$9\frac{3}{4}$	—	31	1	$6\frac{3}{4}$
461 at —	—	$10\frac{1}{4}$	—	19	13	$9\frac{1}{4}$
273 at —	—	$10\frac{1}{2}$	—	11	18	$10\frac{1}{2}$
536 at —	—	$10\frac{3}{4}$	—	24	—	2
910 at —	—	11	—	41	14	2
5391 at —	—	$11\frac{1}{4}$	—	252	14	$—\frac{1}{4}$
372 at —	—	$11\frac{1}{2}$	—	17	16	6
420 at —	—	$11\frac{3}{4}$	—	20	11	3
2163 at 1	7	—	—	171	4	9

Questions.					Answers.		
	s.	d.			l.	s.	d.
364 at 2	5	—	—	—	43	19	8
531 at 3	9	—	—	—	99	11	3
467 at 4	3	—	—	—	99	4	9
205 at 5	11	—	—	—	60	12	11
1734 at 6	$10\frac{1}{2}$	—	—	—	596	1	3
769 at 7	9	—	—	—	297	19	9
134 at 8	$3\frac{1}{2}$	—	—	—	55	8	$3\frac{1}{2}$
410 at 9	$7\frac{1}{2}$	—	—	—	197	14	$9\frac{1}{2}$
416 at 10	$5\frac{1}{2}$	—	—	—	217	10	8
305 at 11	$9\frac{1}{2}$	—	—	—	179	10	$1\frac{1}{4}$
1209 at 12	7	—	—	—	760	13	3
714 at 13	6	—	—	—	481	19	—
197 at 14	$3\frac{1}{2}$	—	—	—	140	15	$5\frac{1}{2}$
251 at 15	7	—	—	—	195	11	5
612 at 16	$4\frac{1}{2}$	—	—	—	501	14	3
247 at 17	9	—	—	—	219	4	3
816 at 18	$2\frac{1}{2}$	—	—	—	742	1	—
970 at 19	8	—	—	—	953	16	8

R U L E III.

If there be pounds in the price, multiply the given quantity by the number of them; and if there be also some odd money, find its produce by the former rules, and add them together.

E X A M P L E S.

Questions.					Answers.		
	l.	s.	d.		l.	s.	d.
213 at 5	—	—	—	—	1065	—	—
708 at 17	—	—	—	—	12036	—	—
1150 at 2	7	6	—	—	2731	5	—
623 at 3	9	—	—	—	2149	7	—

Questions.	l.	s.	d.	Answers.	l.	s.	d.
7 cwt. 2 qr. $15\frac{1}{2}$ lb. at	3	—	7 per cwt.	23	2	9	
3 ton, 5 c. 2 qr. — at	7	9	3 per ton.	24	8	$9\frac{1}{2}$	
17 lb. 5 oz. 14 dwts. at	3	6	9 per lb.	58	6	$5\frac{1}{2}$	
15 lb. 2 oz. 5 dwts. at	4	7	— — — —	66	1	$3\frac{1}{2}$	
7 oz. 15 dwts. 12 gr. at	—	6	3 per oz.	2	8	7	
5 oz. 6 dwts. 17 gr. at	—	5	10 — — —	1	11	$1\frac{1}{2}$	
3 yds. 1 qr. — at	—	17	6 per yd.	2	16	$10\frac{1}{2}$	
4 yds. 2 qr. 3 nls. at	1	2	4 — — —	5	4	$8\frac{1}{2}$	
1 qr. 2 nls. — at	1	12	6 — — —	—	12	$2\frac{1}{2}$	
32 ac. 1 ro. 14 pls. at	1	16	— per acre.	58	4	$1\frac{1}{2}$	
14 ac. 3 ro. 5 pls. at	2	12	10 — — —	39	—	$11\frac{1}{2}$	
3 gal. 5 pints — at	—	7	6 per gal.	1	7	$2\frac{1}{2}$	
12 gal. 3 pts. — at	—	5	8 — — —	3	10	$1\frac{1}{2}$	

R U L E V.

If the price be any even number of shillings; multiply the quantity by half their number, doubling the first figure of the product for shillings, the rest are pounds.

E X A M P L E S.

Questions.	s.	l.	s.
173 at 2	—	17	6
259 at 4	—	51	16
703 at 6	—	210	18
5013 at 8	—	2005	4
872 at 10	—	436	—
460 at 12	—	276	—
627 at 14	—	438	18
598 at 16	—	478	8
214 at 18	—	192	12

R U L E VI.

When the price is any odd number of shillings; work for the greatest even number contained in it by the last rule,

rule, and for the other shilling take $\frac{1}{20}$ th of the given quantity as in rule 1. Or, multiply by the number of shillings, and divide the product by 20 to reduce it to pounds.

EXAMPLES.

<i>Questions.</i>				<i>Answers.</i>	
	s.			l.	s.
732 at 3	—	—		109	16
147 at 7	—	—		51	9
371 at 9	—	—		166	19
586 at 11	—	—		322	6
240 at 13	—	—		156	—
652 at 15	—	—		489	—
897 at 17	—	—		762	9
1046 at 19	—	—		993	14

RULE VII.

If there be a fraction in the given quantity; after having worked for the integral part by any of the former rules, find the produce of the fraction by multiplying the price by the numerator and dividing the product by the denominator, then add them together for the answer.

EXAMPLES.

<i>Questions.</i>				<i>Answers.</i>		
	l.	s.	d.	l.	s.	d.
273 $\frac{1}{4}$ at — 2 6	—	—		34	3	1 $\frac{1}{2}$
751 $\frac{1}{2}$ at 2 17 10	—	—		2173	1	9
530 $\frac{1}{4}$ at — 14 —	—	—		371	10	6
178 $\frac{1}{8}$ at — 17 —	—	—		151	12	4 $\frac{1}{2}$
231 $\frac{1}{8}$ at — 7 9 $\frac{1}{2}$	—	—		90	4	8 $\frac{1}{4}$
762 $\frac{1}{2}$ at 1 12 6	—	—		1239	4	6
817 $\frac{1}{10}$ at 3 7 4	—	—		2751	11	6 $\frac{1}{2}$

TARE

TARE and TRET.

GROSS *weight* of any commodity, is its own weight together with that of its package, whether cask, chest, or whatever else.

Tare is the weight of the package, or an allowance made instead of it.—What remains after the tare is taken from the gross, may be called *tare-futtle*, if there be more deductions.

Tret is an allowance of 4 *lb.* upon every 104 *lb.* of tare-futtle on account of dust or other waste.—What remains after tret is deducted, may be called *tret-futtle*, if there be any following deduction.

Cloff is an allowance of 2 *lb.* for every 3 *cwt.* and some say for every 100 *lb.* of tret-futtle, to make the weight hold good when sold by retail.

When all the deductions are made, the last remainder is called *neat* or *net* weight.

Note 1. When the tare is at so much per *cwt.* it will be best to divide it into aliquot parts of it, like as in the rule of practice.

2. The tret being 4 to 104, or 1 to 26, will be found by taking the 26th part of the tare-futtle.

3. In calculating oil and spirits, $7\frac{1}{2}$ *lb.* neat are allowed to the gallon.

EXAMPLES.

1. Gross 17 *cwt.* 3 *qr.* 14 *lb.* tare 12 *lb.* per *cwt.* tret 4 to 104, and cloff 2 to 100 or 1 to 50. How much neat?

	<i>cwt. qr. lb.</i>			
<i>lb.</i>	17	3	14	gross.
$8 = \frac{1}{14}$	1	1	3	
$4 = \frac{1}{2}$	—	2	$15\frac{1}{2}$	
	1	3	$18\frac{1}{2}$	tare.
26	15	3	$23\frac{1}{2}$	tare-futtle.
	—	2	$12\frac{1}{2}$	tret.
50	15	1	$10\frac{1}{4}$	tret-futtle.
	—	1	$6\frac{1}{4}$	cloff.
	15	—	$4\frac{1}{2}$	neat.

2. What

2. What is the neat produce of 30 barrels of anchovies, weighing each 36lb. gross, allowing 8lb. per cent. tare? — — — *Ans.* 993½lb.

3. Gross 12 cwt. 14 lb. tare 1 cwt. 2 qr. 18 lb. how much neat? — — — *Ans.* 10 cwt. 1 qr. 24 lb.

4. Suppose 3 cwt. 1 qr. 5 lb. tare were allowed on 71 cwt. 3 qr. of tobacco; what would be the neat weight? *Ans.* 68 cwt. 1 qr. 23 lb.

5. In five chests of sugar, weighing 112 cwt. 1 qr. gross; how much neat, allowing 12 lb. tare? *Ans.* 111 cwt. 19 lb.

6. In 26 bags of hops, containing 73 cwt. 3 qr. gross, tare 10 lb. per bag; how much neat? *Ans.* 71 cwt. 1 qr. 20 lb.

7. What is the neat weight of 20 barrels of figs, each 3 cwt. 1 qr. 5 lb. gross, tare 14 lb. per Barrel? *Ans.* 63 cwt. 1 qr. 16 lb.

8. In 15 hhd. of tobacco, each 2 cwt. 1 qr. 12 lb. gross, tare 1 qr. 4 lb. per hhd. how much neat? *Ans.* 31 cwt. 8 lb.

9. What is the neat weight of 3 barrels of indigo, each 3 cwt. 2 qr. gross, tare 10½ lb. per cwt? *Ans.* 9 cwt. 2 qr. 1½ lb.

10. What is the neat weight of 4 hhd. of sugar, weighing as under,

	c.	q.	lb.	
gross {	3	2	14	} tare of the whole 1 cwt. 3 qr. 5 lb.
	5	1	7	
	2	3	18	
	1	3	26	

Ans. 12 cwt. 4 lb.

11. Five casks of raisins, wt. viz.

	cwt.	qr.	lb.	lb.	
Nº 1.	3	2	12	tare	18
2.	2	3	9	—	16
3.	4	1	17	—	23
4.	5	-	8	—	27
5.	1	3	20	—	14

} how much neat? *Ans.* 16 cwt. 3 qr. 24 lb.

12. What is the neat weight of the three following lots of wormfeed? viz.

cwt. qr. lb.

N^o 1. 3 2 8 — tare 12 lb. each.

2. 2 3 26

3. 3 1 15

Anf. 9 cwt. 2 qr. 13 lb.

13. In 15 cwt. 3 qr. 14 lb. gross; tare 13 lb. per cwt. and tret 4 lb. per 104 lb. how much neat?

Anf. 13 cwt. 1 qr. 27½ lb.

14. Suppose 17½ lb. per cwt. tare, and 4 lb. per 104 lb. tret, were allowed on 7 casks of prunes, each 3 cwt. 1 qr. 5 lb. gross; what would be the neat weight?

Anf. 18 cwt. 2 qr. 24 lb.

15. What is the neat weight of 3 bbls. of sugar, weighing as follows: the first, 4 cwt. 5 lb. gross, tare 73 lb. the second, 3 cwt. 2 qr. gross, tare 56 lb. and the third, 2 cwt. 3 qr. 17 lb. gross, tare 47 lb. allowing also 4 lb. per 104 lb. tret? — *Anf.* 8 cwt. 2 qr. 4 lb.

16. In 4 casks of currants, each 7 cwt. 1 qr. 12 lb. gross; tare 2 qr. 10 lb. per cask, tret 4 lb. per 104 lb. and cloff 2 lb. per 100 lb. how much neat?

Anf. 25 cwt. 2 qr. 1½ lb.

17. In 23 cwt. 3 qr. 7 lb. gross; how much neat, allowing 1 qr. 3 lb. per cwt. tare, 4 lb. per 104 lb. tret, and 2 lb. per 300 lb. cloff? — *Anf.* 16 cwt. 1 qr. 22½ lb.

18. In 17 cwt. 17 lb. gross weight of galls; how much neat, allowing 18 lb. per cwt. tare, 4 lb. per 104 lb. tret, and 2 lb. per 3 cwt. cloff? — *Anf.* 13 cwt. 3 qr. 1½ lb.

19. In three casks of oil, weighing as follows: N^o 1. 3 cwt. 17 lb. N^o 2. 2 cwt 3 qr. 5 lb. N^o 3. 4 cwt. 1 qr. 17 lb. how many gallons, allowing 18 lb. per cwt. tare, and 7½ lb. neat to a gallon? — *Anf.* 129½ gal.

20 In 7 casks of oil, each weighing 3 cwt. 1 qr. gross; how many neat gallons, allowing 20 lb. per cwt. tare, and 7½ lb. per gallon? — *Anf.* 297½ gal.

BILLS.

BILLS of PARCELS and BOOK-DEBTS, &c. 75
BILLS OF PARCELS and
BOOK-DEBTS, &c.

M^{R.} James Elford,
 Bought of William Woollendraper.
 Newcastle, 2d of March, 1763, s. d.

15 yds. of fine broad cloth	at	13	6	per yd.
24 ——— superfine ditto	—	18	9	—
27 ——— yard-wide ditto	—	8	4	—
16 ——— drugget	—	6	3	—
12 ——— serge	—	2	10	—
32 ——— shalloon	—	1	8	—

£. 53 4 10

Mr. Nicholas Norton,
 Bought of Henry Hofier.
 London, 24th of March, 1763. s. d.

9 pair of worsted stockings	at	4	6	per pair.
6 ——— filk ditto	—	15	9	—
17 ——— thread	—	5	4	—
23 ——— cotton	—	4	10	—
14 ——— yarn	—	2	4	—
18 ——— women's filk gloves	—	4	2	—
19 yds. of flannel	—	1	7½	per yd.

£. 23 15 4½

Mr. Matthew Milton,
 Bought of Leonard Linendraper, and Co.
 Durham, 9th of April, 1763. s. d.

40 ells of dowlas	at	1	6	per ell.
34 ——— diaper	—	1	4½	—
31 ——— holland	—	5	8	—
29 yds of irish cloth	—	2	4	per yd.
17½ ——— muslin	—	7	2½	—
13½ ——— cambric	—	10	6	—
27 ——— printed linen	—	2	5	—

£. 34 5 10½

76 *BILLS of PARCELS and BOOK-DEBTS, &c.*

The honourable the lady Strawberry,

To Miles Mercer, Dr

York, 1763.

s. d.

April 12th, 9½ yds of silk — at 12 9 per yd.

— 27th, 13 — flowered ditto 15 6 —

June 18th, 11¼ — lustring — '6 10 —

— — 14 — brocade — 11 3 —

July 22d, 12½ — fatten — 10 8 —

— 30th, 11½ — velvet — 18 — —

£. 44 15 10

Samuel Simpson, Esq;

To George Grocer, Dr.

London, 1763.

s. d.

July 18th, 15½ lb. of currants — at - 4 per lb.

— — 17¼ — malaga raisins - 5½ —

— — 19¾ — raisins of the sun - 6 —

Aug. 10th, 17 — rice — - 3½ —

— 13th, 8¼ — pepper — 1 6 —

Sept. 14th, 3 sugar loaves, wt. 32½ lb. 8½ —

— 21st, 13 oz. of cloves — 9 per oz.

£. 3 13 — ½

Tristram Shandy, Esq;

Bought of William Winecooper.

6th of May, 1763.

s. d.

Palm-sack — 12 gallons, at 8 6 per gal.

Port, red — 17 — — 5 8 —

Claret — 9 — — 8 9 —

Lisbon, white — 34 — — 4 10 —

Rhenish — 22½ — — 6 4 —

Sherry — 27¾ — — 6 2 —

£. 37 15 — ½

Mrs.

BILLS of PARCELS and BOOK-DEBTS, &c. 77

Mrs. March.

To Mary Millener, Dr.

1763, Bristol.

s. d.

May 3d, Silver ribbon, 21 yds. at 2 2 per yd.

— — Fine lace — 11½ yds. — 10 6 —

— 7th, Sarcenet hoods, 8 — — 4 3 —

June 10th, India Fans, 17 — — 3 10 —

— 12th, Kid gloves, 9 pair — 2 2 p. pair.

July 6th, Lambs ditto, 5 doz. — 1 2 —

— 7th, Bobbin — 12 pcs. — — 6 p. pce.

£. 18 — 11

Mr. Roger Retail,

Bought of Thomas Teapot and Co.

June 3d, 1763.

s. d.

24½ lb. of royal green tea, at 18 6 per lb.

21½ — imperial tea, — 24 — —

35½ — best bohea, — 13 10 —

17½ — coffee, — 5 4 —

25 — double refin'd sugar, 1 1½ —

9 sugar loaves, wt. 137 lb. — 7½ —

£. 83 5 9

Capt. James Dixon,

Bought of Christopher Cornchandler.

15th of July, 1763. l. s. d.

Wheat, 7 qr. 3 bush. at 1 8 - per qr.

Rye, 9 — 7 — — 1 1 6 —

Oats, 17 — 4 — — 10 8 —

Peas, 12 bush. — — — 2 9 per bush.

Beans, 9 — — — 3 5 —

Malt, 17 — — — 4 8 —

Hops, 25 lb. — — — 1 4 per lb.

£. 39 1 10½

78 *BILLS of PARCELS and BOOK-DEBTS, &c.*

Mr. *Conrade Compound*, Bought of *Daniel Druggist*.
London, 17th of Aug. 1763.

			s.	d.
Cochineal,	—	21 $\frac{1}{4}$ lb. at	29	6 per lb.
Opium,	—	6 $\frac{1}{4}$ —	6	4 —
Scammony,	—	53 $\frac{1}{4}$ —	8	10 —
Contrayerva,	—	14 $\frac{1}{4}$ —	17	— —
Galls,	—	93 —	—	10 —
Gum arabic	—	71 $\frac{1}{4}$ —	1	2 $\frac{1}{2}$ —
Sassafras,	—	122 —	—	4 $\frac{1}{2}$ —

£. 80 1 0 $\frac{1}{2}$.

Sir *Jeffery Slingstone*, Bought of *Samuel Silversmith*.
Sept. 8th, 1763. oz. dwt. gr. s. d.

A punch-bowl,	wt.	23	4	—	at	5	10	per oz.
A tankard,	—	10	3	6	—	6	2	—
A tea-pot and lamp,		30	5	12	—	7	3	—
6 plates	—	73	11	5	—	6	1	—
18 spoons,	—	41	—	10	—	6	3	—

£. 56 1 4.

Mr. *George Davis*,

Bought of *Champion Cheesemonger*.

Sept. 6, 1763. cwt. qr. lb. l. s. d.

13 cheshire cheeses,	wt.	5	3	12	at	1	12	6 per cwt.
25 glocester ditto,	—	3	—	18	—	1	8	—
47 stilton ditto,	—	1	2	5	—	2	4	8 —
17 lb. of cream ditto,	at	7 $\frac{1}{2}$ d.	per lb.	—	—	—	—	—
9 flitches of bacon	wt.	53	ft.	3	lb.	at	4s. 8d.	per ft.
15 $\frac{1}{2}$ firkins of butter,	at	1	l.	8s.	each.	—	—	—

£. 52 - 9 $\frac{1}{2}$

Mr.

Mr Timothy Tradewell,

Bought of Simon Screw, 5 casks of sugar.

London, Sept. 16, 1763.

	cwt.	q.	lb.	qr.	lb.
N ^o 25.	7	3	17	tare	3 14 each.
26.	8	2	11		
27.	8	-	22		
28.	7	2	5		
29.	8	1	16		

Gr.

Tr.

Nt. at 2l. 6s. 8d. per cwt. £. 84 12 1

Sir Gilbert Gosling,

Bought of David Jenkins, 5 butts of oil.

Oct. 12, 1763.

	cwt.	qr.	lb.	
N ^o 1.	9	3	27	} tare 18 lb. per cwt.
2.	10	2	11	
3.	11	1	6	
4.	10	3	14	
5.	12	2	18	

Gr.

Tr.

Nt. at 24l. 10s. per tun. £. 72 2 5 $\frac{1}{2}$

Note, 7 $\frac{1}{2}$ lb. make a gallon, and 236 gallons are a tun.

SIMPLE INTEREST.

INTEREST is the premium allowed for the loan of money.

The sum lent is called the *principal*.

The sum of the principal and interest is called the *amount*.

Interest is allowed at so much *per cent. per annum*, which premium *per cent. per annum*, or interest of 100 *l.* for a year, is called the *rate* of interest.

Interest is of two sorts, *simple* and *compound*.

Simple interest is that which is allowed for the principal lent only.

Note, The rules for simple interest serve also to calculate factorage, brokerage, insurance, stocks, or any thing else rated at so much per cent.

R U L E S.

I. *To find the interest for a year*, multiply the principal by the rate, and divide the product by 100.

Note, When the rate is one, or can be divided into several convenient aliquot parts of 100, take the same part or parts of the principal for the interest of a year.

E X A M P L E S.

1. What is the interest of 450 *l.* for a year at 5 *per cent. per annum*? — — *Ans.* 22*l.* 10*s.*

2. What is the interest of 230 *l.* 10*s.* at 4 *per cent. per annum*? — — *Ans.* 9*l.* 4*s.* 4*d.* 3 $\frac{1}{4}$ *q.*

3. What is the interest of 715*l.* 12*s.* 6*d.* at 4 $\frac{1}{2}$ *per cent. per annum*? — — *Ans.* 32*l.* 4*s.* 0 $\frac{1}{4}$ *d.*

II. *To find the interest for several years*, multiply the interest of one year by the number of them.

Note, When there are several years, or several years with some parts of a year, it is commonly best to multiply them by the rate, and divide the product into aliquot parts of 100, taking the same parts of the principal for the answer.

EXAMPLES.

1. What is the interest of 720*l.* for 3 years, at 5 *per cent. per annum*? — — — *Ans.* 108 *l.*

2. What is the interest of 355*l.* 15*s.* for 4 years, at 4 *per cent. per annum*? — — — *Ans.* 56*l.* 18*s.* 4*d.* 3 $\frac{1}{4}$ *q.*

3. What is the interest of 32*l.* 5*s.* 8*d.* for 7 years, at 4 $\frac{1}{4}$ *per cent. per annum*? — — — *Ans.* 9*l.* 12*s.* 1 $\frac{3}{10}$ *d.*

III. *If there be any parts of a year, as $\frac{1}{2}$, $\frac{1}{4}$, $\frac{3}{4}$, &c. the interest for them is found by taking the same parts of one year's interest, when it is not convenient to use the note in the last case.*

EXAMPLES.

1. What is the interest of 170*l.* for 1 $\frac{1}{2}$ year, at 5 *per cent. per annum*? — — — *Ans.* 12*l.* 15*s.*

2. What is the interest of 205*l.* 15*s.* for $\frac{1}{2}$ of a year, at 4 *per cent. per annum*? — — — *Ans.* 2*l.* 1*s.* 1*d.* 3 $\frac{1}{4}$ *q.*

3. What is the interest of 319*l.* 6*d.* for 5 $\frac{1}{4}$ years, at 3 $\frac{3}{4}$ *per cent. per annum*? — — — *Ans.* 68*l.* 15*s.* 9*d.* 2 $\frac{7}{8}$ *q.*

IV. *If there be any number of months under 12, divide them into aliquot parts of a year, and work for them as in the last case.*

EXAMPLES.

1. What is the interest of 105*l.* for 4 months, at 5 *per cent. per annum*? — — — *Ans.* 1*l.* 15*s.*

2. What is the interest of 210*l.* 17*s.* for one year and 9 months, at 4 $\frac{1}{2}$ *per cent. per annum*?
Ans. 16*l.* 12*s.* 1 $\frac{13}{100}$ *d.*

3. What is the interest of 312*l.* 10*s.* 4*d.* for 3 years and 8 months, at 3 $\frac{1}{4}$ *per cent. per annum*?
Ans. 37*l.* 4*s.* 9*d.* 3 $\frac{68}{100}$ *q.*

V. *For any number of weeks, if they be no part or parts of 52, multiply the interest of a year by them and divide by 52.*

And for any number of days, multiply the interest of a year by them and divide by 365.

A TABLE *showing the number of days from any day of one month to the same day of any other month.*

From	Jan.	Feb.	Ma.	Ap.	May	Jun.	July	Aug.	Sep.	Oct.	Nov.	Dec.
Jan.	365	334	306	275	245	214	184	153	122	92	61	31
Feb.	31	365	337	306	276	245	215	184	153	123	92	62
Mar.	59	28	365	334	304	273	243	212	181	151	120	90
Apr.	90	59	31	365	335	304	274	243	212	182	151	121
May	120	89	61	30	365	334	304	273	242	212	181	151
June	151	120	92	61	31	365	335	304	273	243	212	182
July	181	150	122	91	61	30	365	334	303	273	242	212
Aug.	212	181	153	122	92	61	31	365	334	304	273	243
Sept.	243	212	184	153	123	92	62	31	365	335	304	274
Oct.	273	242	214	183	153	122	92	61	30	365	334	304
Nov.	304	273	245	214	184	153	123	92	61	31	365	335
Dec.	334	303	275	244	214	183	153	122	91	61	30	365

Note, In leap year, if the end of the month of february be in the time, one day more must be added on that account.

EXAMPLES.

1. What is the interest of 300*l.* for 14 weeks, at 5 per cent. per annum? — — — *Ans.* 4*l.* 9*½**d.*

2. What is the interest of 212*l.* for 2 years 17 weeks, at 4 per cent. per annum? — *Ans.* 19*l.* 14*s.* 7*d.* 3*¼**d.*

3. What is the interest of 107*l.* for 117 days, at 4½ per cent. per annum? — — — *Ans.* 1*l.* 12*s.* 7*⅞**d.*

4. What is the amount of 120*l.* from jan. 7, to sept. 12, 1763, at 4 per cent per annum?

Ans. 123*l.* 5*s.* 2*d.* 2*⅞**d.*

5. What

5. What is the interest of 213*l.* from feb. 12, to june 5, 1764, it being leap year, at $3\frac{1}{2}$ per cent per annum?

Ans. 2*l.* 6*s.* 6*d.* $3\frac{101}{1111}$.*q.*

N. B. The following neat and concise table and method for working simple interest for any number of days, at any rate of interest, is taken from the *gentleman's diary*, I having only continued the table into decimal numbers.

Numb.	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>	Nu.	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>	Nu.	<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
1000000	2739	14	6	0.99	3000	8	4	4	2.41	5	0	0	3	1.15
900000	2465	15	0	3.29	2000	5	9	7	0.27	4	0	0	2	2.52
800000	2191	15	7	1.59	1000	2	14	9	2.14	3	0	0	1	3.89
700000	1917	16	1	3.89	900	2	9	3	3.12	2	0	0	1	1.26
600000	1643	16	8	2.19	800	2	3	10	0.11	1	0	0	0	2.63
500000	1369	17	3	0.49	700	1	18	4	1.10	0.9	0	0	0	2.37
400000	1095	17	9	2.79	600	1	12	10	2.08	0.8	0	0	0	2.10
300000	821	18	4	1.10	500	1	7	4	3.07	0.7	0	0	0	1.84
200000	547	18	10	3.40	400	1	1	11	0.05	0.6	0	0	0	1.58
100000	273	19	5	1.70	300	0	16	5	1.04	0.5	0	0	0	1.32
90000	2	6	11	6.033	200	0	10	11	2.03	0.4	0	0	0	1.05
80000	219	3	6	2.96	100	0	5	5	3.01	0.3	0	0	0	0.75
70000	191	15	7	1.59	90	0	4	11	0.71	0.2	0	0	0	0.53
60000	164	7	8	0.22	80	0	4	4	2.41	0.1	0	0	0	0.26
50000	136	19	8	2.85	70	0	3	10	0.11	0.09	0	0	0	0.24
40000	109	11	9	1.48	60	0	3	3	1.81	0.08	0	0	0	0.21
30000	82	3	10	0.11	50	0	2	8	3.51	0.07	0	0	0	0.18
20000	54	15	10	2.74	40	0	2	2	1.21	0.06	0	0	0	0.16
10000	27	7	11	1.37	30	0	1	7	2.90	0.05	0	0	0	0.13
9000	24	13	13	2.23	20	0	1	1	0.60	0.04	0	0	0	0.11
8000	21	18	4	1.10	10	0	0	6	2.30	0.03	0	0	0	0.08
7000	19	3	6	2.96	9	0	0	5	3.67	0.02	0	0	0	0.05
6000	16	8	9	0.82	8	0	0	5	1.04	0.01	0	0	0	0.03
5000	13	13	11	2.68	7	0	0	4	2.41					
4000	10	19	2	0.55	6	0	0	3	3.78					

R U L E.

Multiply the principal by the rate, both in pounds; multiply the product by the number of days, and divide this last product by 100; then take from the table the several sums which stand opposite the several parts of the quotient, and adding them together will give the interest required.

E x.

EXAMPLES.

1. What is the interest of 225*l.* 10*s.* for 23 days, at $4\frac{1}{2}$ per cent. per annum?

Then in the table

princ.	225.5		<i>l.</i>	<i>s.</i>	<i>d.</i>	<i>q.</i>
rate	4.5	against 200 is	0	10	11	2.03
		30 —	0	1	7	2.90
	1014.75	3 —	0	0	1	3.89
days	23	0.3 —	0	0	0	0.79
		0.09 —	0	0	0	0.24
<hr/>			<hr/>			
100)	23339.25		Ans.	0	12	9 1.85 true in
	<hr/>					the last place of decimals.
	233.3925					

2. What is the interest of 17*l.* 5*s.* for 117 days, at $4\frac{1}{2}$ per cent. per annum? ——— Ans. 5*s.* 3*d.* 0.12*q.*

3. What is the interest of 112*l.* 12*s.* 6*d.* from the 8th of may, to the 3d of november, at 4 per cent. per annum?
Ans. 2*l.* 4*s.* 2*d.* 0.92*q.*

QUESTIONS

Concerning *brokerage*, *factorage*, *insurance*, and *Stocks*.

Brokerage is the allowance made to brokers for assisting others in buying or disposing of their goods.

Factorage, *provision*, or *commission* is an allowance made to factors or agents beyond sea, for buying or selling of goods for their employers.

Insurance is security given, in consideration of a premium paid down, to restore, to a certain value for which the premium is advanced, the loss or damage on ships, houses, goods, &c. by storms, fire, &c. But in the calculations, the word *insurance* is commonly wrote for premium.

Stocks are the public funds of the nation; the shares of which being transferable from one person to another, occasion the extensive business, or worst kind of gaming, called stock-jobbing.

EXAMPLES.

1. What is the brokage of 610*l.* at 5*s.* or $\frac{1}{4}$ per cent?
Ans. 1*l.* 10*s.* 6*d.*

2. What is the brokage of 372*l.* 7*s.* 4*d.* at 4*s.* 6*d.* per cent? ——— *Ans.* 16*s.* 9 $\frac{1}{3}$ ⁹*d.*
3. What is the factorage of 920*l.* at 3 $\frac{1}{2}$ per cent? *Ans.* 32*l.* 4*s.*
4. What is the commission of 508*l.* 17*s.* 10*d.* at 1 $\frac{1}{2}$ per cent? ——— *Ans.* 7*l.* 12*s.* 8 $\frac{1}{10}$ ⁰*d.*
5. What is the insurance of 900*l.* at 10 $\frac{1}{4}$ per cent? *Ans.* 96*l.* 15*s.*
6. What is the insurance of 712*l.* 6*s.* for 8 months, at 7 $\frac{1}{2}$ per cent. per annum? — *Ans.* 35*l.* 12*s.* 3*d.* 2 $\frac{3}{4}$ *q.*
7. What is the purchase of 1200*l.* south-sea stock at 103 $\frac{1}{4}$ per cent? ——— *Ans.* 1243*l.* 10*s.*
8. What is the purchase of 912*l.* 14*s.* bank stock, at 127 $\frac{1}{4}$ per cent? ——— *Ans.* 1165*l.* 19*s.* 5*d.* 3 $\frac{1}{2}$ ⁷*q.*
9. What is the purchase of 2380*l.* india stock, at 147 $\frac{1}{2}$ per cent? ——— *Ans.* 3504*l.* 11*s.*
10. What is the purchase of 816*l.* 12*s.* bank annuities, at 89 $\frac{1}{4}$ per cent? ——— *Ans.* 729*l.* 16*s.* 8*d.* 2 $\frac{1}{4}$ ⁷*q.*

COMPOUND INTEREST.

COMPOUND interest is that which is allowed, not only for the sum lent, but also for its interest as it becomes due at the end of each stated time of payment.

R U L E S.

I. Find the amount of the given principal for the time of the first payment, by simple interest; then consider this amount as the principal for the second payment, whose amount calculate in the same manner; and so on through all the payments, still accounting the last amount as the principal of the next payment. *Or,*

II. Find the amount of one pound for the time of the first payment, and multiply it by itself so often as are the number of payments wanting 1, that is, twice by itself if there be three payments, thrice if there be four, &c. then the last product multiplied by the principal gives the whole amount.

Note, It is not necessary that the payments should be yearly, for the rule will hold whether they be yearly, half yearly, quarterly, monthly, or any other aliquot part of a year; but there must be a complete integer number of the times of payments, not a certain number of times and part of another, for the rule takes no notice of such parts, nor will it be just to calculate for a complete time and take the same part of the result as is the part of the time; but in this manner has Shepherd falsely calculated some of his examples. It is possible to perform all such calculations, both parts of times and whole ones, without logarithms, though the trouble is, in some cases, intolerable: but by the logarithms it is as easy to perform the calculations with parts of times of payments, as with whole ones.

E X A M P L E S.

1. What will 50*l.* amount to in 5 years, at 5 *per cent. per annum*, compound interest? — *Ans.* 63*l.* 16*s.* 3 $\frac{1}{4}$ *d.*
2. What will 50*l.* supposing the interest payable half-yearly, amount to in 5 years, or 10 half-years, at 5 *per cent. per annum*, compound interest? — *Ans.* 64*l.* 1*d.*
3. What will 50*l.* the interest payable quarterly, amount to in 5 years, at 5 *per cent. per annum*, compound interest? — *Ans.* 64*l.* 2*s.* 0 $\frac{1}{4}$ *d.*
4. What is the compound interest of 370*l.* forborn 6 years, at 4 *per cent per annum*? — *Ans.* 98*l.* 3*s.* 4 $\frac{1}{4}$ *d.*
5. What is the compound interest of 410*l.* forborn 2 $\frac{1}{2}$ years, at 4 $\frac{1}{2}$ *per cent. per annum*, supposing the interest payable half-yearly? — *Ans.* 48*l.* 4*s.* 11 $\frac{1}{4}$ *d.*
6. What is the amount of 217*l.* forborn 2 $\frac{1}{4}$ years, at 5 *per cent. per annum*, supposing the interest payable quarterly? — *Ans.* 242*l.* 13*s.* 4 $\frac{1}{2}$ *d.*

REBATE or DISCOUNT.

REBATE or *discount*, is the difference between a sum of money due at a certain time to come, and its present worth.

The *present worth* of any sum or debt, due some time hence, is such a sum, as if put to interest, would, in the time and at the rate for which the discount is to be made, amount to the sum or debt then due.

R U L E.

R U L E.

As the amount of 100*l.* for the given rate and time,
Is to 100*l.* or the interest of 100*l.* for the given time;
So is the given sum or debt,
To the present worth, or discount of the given sum.

Note 1. The meaning of four things wrote in the form above, is that they are the four terms of a rule of three questions.

2. "The method used among bankers, &c. in discounting bills, is to find the interest of the sum drawn for, from the time the bill is discounted to the time when it becomes due, (including the days of grace) which interest they reckon as the discount, thereby making the sum more than it really is."

But when goods are bought or sold, and discount is to be made for present payment, at any rate per cent. without regard to time, the interest of the sum as calculated for a year is the discount.

E X A M P L E S.

1. What is the present worth of 700*l.* due 9 months hence, discount at 5 per cent. per annum?

Ans. 674*l.* 13*s.* 11*d.* 2 $\frac{7}{8}$ ⁹*q.*

2. What is the discount of 312*l.* for 6 months, at 6 per cent. per annum? — *Ans.* 9*l.* 1*s.* 8*d.* 3 $\frac{9}{16}$ ⁹*q.*

3. What is the rebate of 125*l.* 10*s.* payable 15 months hence, at 4 $\frac{1}{2}$ per cent. per annum?

Ans. 4*l.* 10*s.* 8*d.* 2 $\frac{5}{8}$ ⁸*q.*

4. What is 217*l.* 4*s.* 6*d.* due 5 months hence, worth in present money, discount at 5 $\frac{1}{2}$ per cent?

Ans. 212*l.* 7*s.* 2 $\frac{14}{19}$ ¹*d.*

5. How much ready money for a note of 73*l.* due 17 months hence, discount at 5 per cent. per annum?

Ans. 68*l.* 3*s.* 5 $\frac{23}{27}$ ³*d.*

6. Sold goods to the amount of 83*l.* 6*s.* to be paid 6 months hence; what must I have in present money, discount at 8 per cent. per annum? — *Ans.* 80*l.* 1*s.* 11 $\frac{1}{3}$ ¹*d.*

7. Bought goods to the value of 35*l.* 8*s.* to be paid 8 months hence; what must I pay in present money, discount at 7 per cent. per annum? *Ans.* 33*l.* 16*s.* 5 $\frac{1}{3}$ ¹*d.*

8. If a legacy of 600*l.* is left me on the 3d of may, 1763, to be paid on the christmas-day following; what

must I receive, when I allow 5 per cent. per annum discount for present payment? *Ans.* 581*l.* 4*s.* 2*d.* $1\frac{1}{3}\frac{1}{4}$ q.

9. What is the present worth of 60*l.* payable at two 3 months, at 5 per cent. per annum discount?

Ans. 58*l.* 17*s.* 11*d.* $2\frac{1}{10}\frac{1}{4}$ q.

10. What is the present worth of 120*l.* payable as follows, viz. 50*l.* at 3 months, 50*l.* at 5 months, and the rest at 8 months, discount at 6 per cent. per annum?

Ans. 117*l.* 5*s.* $5\frac{1}{4}$ d.

EQUATION OF PAYMENTS.

EQUATION of payments is the finding a time, when if a sum of money be paid which is equal to the sum of several others due at different times, no loss will be sustained by either party.

*The * common rule is to*

Multiply each payment by the time it is due at, then dividing the sum of the products by the sum of the payments, the quotient is the equated time.

EXAMPLES.

1. There is a debt of 60*l.* to be paid 30*l.* at 2 months, and

* The only true rule is this:

If p be the first payment, and t the time till it be due; also $P =$ any other payment, and $T =$ its time; moreover if r be one year's

interest of 1*l.* put $a = T + t + \frac{P + p}{pr}$, and $c = Tr + \frac{PT + pt}{pr}$;

then $\frac{a \pm \sqrt{a^2 - 4c}}{2}$ is the equated time for these two payments

then by the same rule an equated time may be found for this and a third payment, &c.

This rule is the ingenious Mr. Malcolm's: *Kersey's* rule, which *Dilworth* has given in pag. 143 of his *Schoolmaster's assistant* as the true method of working equation of payments, is false.

and 30*l.* at 4 months; but if it be reduced to one payment, at what time must it be made?—*Ans.* at 3 months.

2. A debt of 120*l.* due as follows, viz. 50*l.* at 2 months, 40*l.* at 5 months, and the rest at 7 months: when must the whole be paid? — *Ans.* at 4 $\frac{1}{4}$ mo.

3. A debt of 500*l.* is to be discharged thus, viz. 100*l.* present; 300*l.* at 4 months, and the rest at 6 months: what is the equated time for the whole? — *Ans.* 3 $\frac{3}{5}$ mo.

4. A debt is to be discharged by paying $\frac{1}{3}$ at 3 months, $\frac{1}{3}$ at 5 months, and the rest at 6 months: what is the equated time for the whole? — *Ans.* 4 $\frac{1}{6}$ mo.

5. A debt is to be discharged thus, viz. $\frac{1}{4}$ present, and $\frac{1}{4}$ every 3 months after, till the whole be discharged: what is the equated time for the whole? — *Ans.* 4 $\frac{1}{2}$ mo.

SINGLE FELLOWSHIP.

SINGLE Fellowship is a rule by which any number may be divided into any assigned number of parts, which shall be proportional to so many other proposed numbers, each to each.

By this rule are adjusted the gain or loss or charges of merchants in company, the effects of bankrupts, legacies in case of a deficiency of assets, &c.

R U L E.

Make the sum of the numbers, to which the required parts must be proportional, the first term; the number to be parted or divided, the second; and each of the given numbers, to which the required ones must be proportional, the several third terms of so many rule-of-three questions; the fourth terms of which will be the respective parts required.

Note 1. The first and third contractions of the rule-of-three are the best for working questions in this rule; because the two first terms of all the statings being the same, there will be had a constant multiplier or divisor for the third terms.

2. When two or more of the terms, to which the required ones must be proportional, are equal; so many operations will be saved as there are equalities.*

E X A M P L E S.

1. Divide the number 120 into 3 such parts as shall be to each other as 1, 2, and 3. — *Ans.* 20, 40, and 60.

2. Two merchants, *A* and *B* trade together; *A* puts into the stock 60*l* and *B* puts in 40*l*. and gain by trading 24*l*. what are their shares of it?

Ans. *A*'s share is 14*l*. 8*s*. and *B*'s 9*l*. 12*s*.

3. Two merchants, *C* and *D*, made a stock of 120*l*. whereof *C* contributed 75*l*. by trading they lost 30*l*. what must each sustain of it? — *Ans.* *C* 18*l*. 15*s*. and *D* 11*l*. 5*s*.

4. Three merchants, whose stock is 700*l*. whereof *E* contributed 123*l*. *F* 358*l*. and *G* the rest, gain by trading 125*l*. 10*s*. what must each have of it?

Ans. *E* must have 22*l*. 1*s*. 0*d*. $2\frac{3}{11}7$.

F — 64 3 8 $0\frac{3}{11}$

G — 39 5 3 $1\frac{1}{11}$

5. Three merchants, *H*, *I*, and *K*, freighted a ship with 340 tuns of wine, whereof *H* loaded 110 tuns, *I* 97, and *K* the rest; in a storm the seamen were obliged to throw overboard 85 tuns: how much must each sustain of the loss? — *Ans.* *H* $27\frac{1}{2}$, *I* $24\frac{1}{4}$, and *K* $33\frac{1}{4}$ tuns.

6. *A*

* A question in this rule may be proposed and solved in a general manner thus:

To divide the number *n* into *p* parts, which shall be as the numbers *a*, *b*, *c*, &c.

Solution.

$a + b + c + \&c. : n ::$

$$\left\{ \begin{array}{l} a : \frac{an}{a + b + c + \&c.} \\ b : \frac{bn}{a + b + c + \&c.} \\ c : \frac{cn}{a + b + c + \&c.} \\ \&c. \&c. \end{array} \right.$$

6. A piece of ground, consisting of 37 ac. 2 ro. 14 ps. is to be divided among three persons, *L*, *M*, and *N*, in proportion to their estates: now if *L*'s estate be worth 500*l.* a year, *M*'s 320*l.* and *N*'s 75*l.* what quantity of land must each one have?

Ans. *L* must have 20 ac. 3 ro. $39\frac{1}{19}\frac{19}{19}$ ps.

M — 13 1 $30\frac{4}{19}\frac{6}{19}$.

N — 3 - $23\frac{1}{19}\frac{7}{19}$.

7. A person is indebted to *O* 57*l.* 15*s.* to *P* 108*l.* 3*s.* 8*d.* to *Q* 22*l.* 10*d.* and to *R* 73*l.* but at his decease, his effects are found to be worth no more than 170*l.* 14*s.* how must it be divided among his creditors?

Ans. *O* must have 37*l.* 15*s.* 5*d.* $2\frac{5}{19}\frac{3}{19}\frac{0}{19}\frac{2}{19}$ q.

P — 70 15 2 $2\frac{7}{19}\frac{4}{19}\frac{9}{19}\frac{8}{19}$.

Q — 14 8 4 $0\frac{4}{19}\frac{7}{19}\frac{2}{19}\frac{0}{19}$.

R — 47 14 11 $2\frac{1}{19}\frac{3}{19}\frac{5}{19}\frac{8}{19}$.

8. A ship worth 900*l.* being entirely lost, of which $\frac{1}{4}$ belonged to *S*, $\frac{1}{4}$ to *T*, and the rest to *V*; what loss will each sustain, supposing 540*l.* of her were insured?

Ans. *S* will lose 45*l.* *T* 90*l.* and *V.* 225*l.*

9. Four persons, *W*, *X*, *Y*, and *Z*, spent among them 25*s.* and agree that *W* shall pay $\frac{1}{4}$ of it, *X* $\frac{1}{4}$, *Y* $\frac{1}{4}$ and *Z* $\frac{1}{4}$; that is, their shares are to be in proportion as $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{4}$, and $\frac{1}{4}$; what are their shares?

Ans. *W* must pay 9*s.* 8*d.* $3\frac{4}{7}\frac{1}{7}$ q.

X — 6 5 $3\frac{5}{7}\frac{3}{7}$.

Y — 4 10 $1\frac{5}{7}\frac{2}{7}$.

Z — 3 10 $3\frac{1}{7}$.

DOUBLE FELLOWSHIP.

WHEN the shares of partners are continued in company unequal times, they occasion the name *fellowship with time*, or *double fellowship*: which is performed by the following

RULE.

R U L E.*

Multiply each share by the time of its continuance; then divide the gain or loss in proportion to the products, as in single fellowship, by saying, As the sum of the products is to the whole gain or loss, so is each product, to each part of it.

E X A M P L E S.

1. *A* had in company 50*l.* for 4 months, and *B* 60*l.* for 5 months: at the end of which they find 24*l.* gained: how must it be divided between them?

Ans. *A* must have 9*l.* 12*s.* and *B* 14*l.* 8*s.*

2. A ship's company take a prize of 1000*l.* which they agree to divide among them according to their pay and the time they have been on board: now the officers and midshipmen have been on board 6 months, and the sailors 3 months; the officers have 40*s.* a month, the midshipmen 30*s.* and the sailors 22*s.* a month; moreover there are 4 officers, 12 midshipmen, and 110 sailors: what will each man's share be?

Ans. each officer must have 23*l.* 2*s.* 5*d.* 0⁹/₁₁₁¹/₁₁*q.*

— midship. — 17 6 9 3⁶/₁₁₁⁹/₁₁.

— seaman — 6 7 2 0⁸/₁₁₁⁸/₁₁.

3. *A*, *B*, and *C*, have a pasture in common, for which they pay 30*l.* per annum; into which *A* put 7 cows for 3 months, *B* 9 cows for 5 months, and *C* 4 cows for 12 months: what must each pay of the rent?

Ans. *A* must pay 5*l.* 10*s.* 6*d.* 1⁵/₁₁₉⁵/₁₉*q.*

B — 11 16 10 0⁸/₁₁₉⁸/₁₉.

C — 12 12 7 2⁶/₁₁₉⁶/₁₉.

4. *X*,

* Mr. Ward has given an analytical investigation of this rule, and Mr. Malcolm has given the reason of it in a manner evident enough; but I think the most general and elegant manner of proof is thus:

When the times are equal, the shares of the gain or loss are evidently as the stocks, as in single fellowship; and when the stocks are equal, the shares are as the times; wherefore when neither are equal, the shares must be as their products.

4. X, Y, and Z made a joint stock, for 12 months: X at first put in 20*l.* and 4 months after 20*l.* more; Y put in at first 30*l.* at the end of 3 months he put in 20*l.* more, and 2 months after he put in 40*l.* more; Z put in at first 60*l.* and 5 months after he put in 10*l.* more, 1 month after which he took out 30*l.* during the 12 months they gained 50*l.* how much of it must each have?

Ans. X must have 10*l.* 18*s.* 6*d.* $3\frac{4}{8}\frac{1}{4}$.

Y ——— 22 8 1 $0\frac{1}{8}\frac{1}{4}$.

Z ——— 16 13 4 —.

BARTER.

BARTERING is the exchanging of commodities; and as neither party is supposed to sustain any loss, when the commodities exchanged are not of equal value, the defect is supplied with money, &c.

CASE I.

When the quantity of one commodity is given, with its value, or that of its integer; as also the value of the integer, or rate of selling some other commodity to be given for it, to find the quantity of this; or having the quantity of the latter given to find the rate of selling it.

If the amount of the given quantity be unknown, calculate it in the shortest manner you can, from the given value of its integer; then find how much of the other quantity this amount will purchase, at the given rate of selling it; or if the quantity be given, from thence find the rate of selling it.

CASE II.

If the quantities of both commodities, with the rate of selling them be given; to find what quantity of some other commodity or money must be given in case of an inequality of the amounts of the first commodities.

Calculate the amount of each of the two given commodities; then their difference is the money, or amount of the third commodity to be advanced: whose quantity, from thence and its rate, is easily found.

CASE

CASE III.

When, in bartering, one commodity is rated above the ready-money price; to find the quantity and bartering price of the other commodity.

Say, As the ready-money price of the one, is to its bartering price, so is that of the other to its barter price; then the quantity of the latter commodity may be found either from the ready money or bartering prices.

NOTE: These are the most general cases in barter, and such questions as are not contained in them, are easily solved from a little consideration of their nature.

EXAMPLES.

1. How much tobacco at $1l. 16s.$ per cwt. must be given for 3 pipes of wine at $28l. 10s.$ per pipe?

Ans. 47 c. 2 qr.

2. How much cloth at $14s. 6d.$ per yard, must be given for 3 cwt. 3 qr. of sugar at $3l. 4s.$ per cwt?

Ans. 16 yds. $2\frac{6}{8}$ qr.

3. What weight of hops at $1l. 12s.$ per cwt. must be given for 14 cwt. 1 qr. 18 lb. of cheese at $1l. 6s.$ per cwt?

Ans. 11 c. 2 qr. $23\frac{1}{8}$ lb.

4. If 24 yards of cloth be given for 5 cwt. 1 qr. of tobacco, at $1l. 18s.$ per cwt. what is the cloth rated at per yard? ——— *Ans.* 8s. $3\frac{1}{2}$ d.

5. A and B would barter: A hath 40 yards of cloth at $7s. 4d.$ per yard, B has $28\frac{1}{2}$ lb. of tea, at $11s. 6d.$ per lb. whether must pay balance, and how much?

Ans. A must pay $1l. 14s. 5d.$

6. C and D would barter: C has 53 quarters, 5 bush. of corn, at $1l. 10s.$ per quarter; for which D would give 13 cwt. 16 lb. of sugar, at $4l. 12s.$ per cwt. and the ballance in raisins, at $6\frac{1}{2}d.$ per lb. how many lb. of raisins must be given? ——— *Ans.* $737\frac{6}{8}$ lb.

7. E and F barter: E gives to F 90 gallons of brandy, at $7s. 8d.$ per gallon; for which F gives to E 10 guineas in money, and 500 lb. of cotton; what is it valued at per lb. ——— *Ans.* 11d. $2\frac{2}{3}$ q.

8. G and H bartered: G had 13 cwt. 5 lb. of sugar, worth $1l. 15s.$ per cwt. but bartered it with H, at $2l. 4s.$

2l. 4s. per cwt. for wine worth 4s. 8d. per gallon: what was the barter price of the wine, and how much of it was given for the sugar?

Ans. 5s. 10 $\frac{1}{2}$ d. per gal. and 97 $\frac{1}{2}$ $\frac{1}{4}$ gal. equal the sugar.

9. K and L barter: K has woollen cloth worth 8s. per yard, which he barter at 9s. 3d. with L, for linen cloth at 3s. per yard, which is worth only 2s. 7d. per yard: whether has the advantage in barter, and how much linen does L give K for 70 yards of woollen?—*Ans.* 215 $\frac{1}{8}$ yds. of linen; and L has the advantage, his proportional barter price being only 2s. 11d. 3 $\frac{1}{8}$ q.

L O S S and G A I N.

QUESTIONS in this rule are such whose solutions determine the *loss* or *gain* upon commodities; of which questions there are great variety; but may be easily solved from a little consideration and the following proportion, viz. That the *gains* or *losses* are in proportion as their *quantities*, and the contrary.

E X A M P L E S.

1. Bought 5 c. 3 qr. 14 lb. of cheese, at 1l. 12s. per cwt. and sold it again for 2l. 8d. per cwt. what was the gain upon the whole? — *Ans.* 2l. 10s. 11d.

2. If 5 c. 3 qr. 14 lb. be bought for 9l. 8s. and sold for 11l. 18s. 11d. what is the rate of gain per cwt?

Ans. 8s. 8d.

3. If 8 c. 18 lb. be bought for 45l. at what rate per lb. shall I sell it to gain 10l. upon the whole?

Ans. 1s. 2d. 1 $\frac{1}{4}$ $\frac{1}{2}$ q.

4. If 8 c. 18 lb. cost 45l. at what rate per lb. must it be sold, that the loss upon the whole may be 10l?

Ans. 9 $\frac{8}{11}$ $\frac{1}{2}$ d.

5. Bought hops at 4l. 16s. per cwt. at what rate per cwt. must I sell them to gain 15l. per cent?

Ans. 5l. 10s. 4d. 3 $\frac{1}{2}$ q.

6. Bought hops at 4l. 16s. per cwt. at what rate per cwt. must I sell them, to lose 15l. per cent?

Ans. 4l. 1s. 7 $\frac{1}{2}$ d.

7. * If, when I sell cloth at 7s. per yard, I gain 10l. per cent. what will be the gain per cent. when it is sold for 8s. 6d. per yard? ——— *Ans.* 33l. 11s. 5 $\frac{1}{2}$ d.

8. If, when I sell cloth at 7s. a yard, I gain 10l. per cent. what is the gain or loss per cent. when it is sold at 6s. a yard? ——— *Ans.* 5l. 14s. 3 $\frac{1}{2}$ d. loss.

9. If, when I sell sugar at 8s. 6d. a stone, I lose 5l. per cent. what do I gain or lose per cent. when I sell it for 9s. a stone? ——— *Ans.* 11s. 9 $\frac{1}{4}$ d. gain.

10. Bought for 17s. 8d. and sold for 18s. 4d. what was the gain per cent? ——— *Ans.* 3l. 15s. 5d. 2 $\frac{1}{4}$ q.

11. Bought 12 yards of cloth for 5l. 9s. and sold them again at 9s. 6d. a yard: what was the gain or loss per cent? ——— *Ans.* 4l. 11s. 8d. 3 $\frac{1}{8}$ q. gain.

12. At 1 $\frac{1}{2}$ d. per shilling profit, how much per cent? ——— *Ans.* 12l. 10s.

13. At 3s. 6d. to the pound profit, how much per cent? ——— *Ans.* 17l. 10s.

14. Having sold 12 yards of cloth for 5l. 14s. and thereby gained 8l. per cent. what was the prime cost of a yard? ——— *Ans.* 8s. 9d. 2 $\frac{3}{4}$ q.

E X.

* Questions of this sort are seldom rightly understood, or solved: thus, the question beginning at the 7th line of page 33 of the 4th edition of Webster's arithmetic is wrong, the true answer being 35 l. and not 10 l. And in the same manner has Sronchouse falsely solved the 2d example in page 89 of the 3d edition of his arithmetic, the answer to which question should be 23 l. 12 s. 6 d. and not 8 l. 12 s. 6 d. Also in a manner similar to these has Dilworth falsely calculated his 7th example in loss and gain, making the answer 2 l. 16 s. 3 d. instead of 3 l. 1 s. 9 $\frac{1}{4}$ d. though this question has Lowe copied into his book of arithmetic with the false answer; as he has done several others from different authors, whereof one is from Malcolm, who brought out his answer wrong, not by a false method of solution, but by some wrong figures slipping from his pen in the process, or in the proposition. Hill has also run into the same mistake. — The error consists in making the gain or loss of 100 l. the 1d term of the question instead of its amount.

E X C H A N G E.

By exchange is meant the bartering or exchanging of the money of one place for that of another; and, like the bartering of wares, it commonly consists in finding what quantity of the money of one place will be equal to a given sum of another, according to a given course of exchange.

By *course of exchange* is meant the variable sum of the money of one place which is proposed to be given for a constant piece or sum of that of another, to serve for the present, as a rate or proportion by which to exchange other sums, and is sometimes above and sometimes below the *par*.

By the *par of exchange* is meant an intrinsic equality between two pieces or sums of money, one of which is the constant piece or sum to which the *course* is compared.

The money in the banks of foreign ~~places~~ is finer or purer than that which is current in them; and the difference between any sum as valued in the one and its value in the other, is called *agio*.

Note, it is by comparing the bank money with ~~ours~~ that the *par* is ascertained. Also the exchange is always supposed to be made in bank money; and if there be a necessity for taking currency in case of a defect of the bank to answer the bills, the more of it must be received, and that in proportion to the *agio*.

I. With IRELAND, AMERICA, and the WEST-INDIES.

Accounts are kept in Ireland, America, and the West-Indies in pounds, shillings, and pence, as in England; and exchange *per cent.* sterling; the *par* being 108*l.* 6*s.* 8*d.* Irish *per* 100*l.* sterling, or 1*l.* 1*s.* 8*d.* *per* pound: also 5*l.* sterling is accounted worth 7*l.* of the currency of the West-indies, because of the great plenty of foreign coins there; but on the continent of America there is very little coin of any sort circulating.

E X A M P L E S.

1. London remits to Dublin 375*l.* 15*s.* what must be received there, exchange at 110 *per cent*?

Ans. 413*l.* 6*s.* 6*d.*

2. Dublin remits to London 413*l.* 6*s.* 6*d.* what must be received there, exchange at 110 *per cent*?

Ans. 375*l.* 15*s.*

3. Lon-

3. London remits to Jamaica for 212*l.* 12*s.* 6*d.* sterling: what must be received for it, exchange at 135 *per cent*? — — — — — *Ans.* 287*l.* 10½*d.*

4. Jamaica remits to London for 287*l.* 10½*d.* currency: what must be received for it, exchange at 135 *per cent*? — — — — — *Ans.* 212*l.* 12*s.* 6*d.*

II. With HOLLAND, FLANDERS, and GERMANY.

In these places accounts are kept, sometimes in pounds, shillings, and pence, as in England; and sometimes in guilders, stivers, and pennings; the money of *Holland* and *Flanders* is distinguished by the name *flemish*, and they exchange by the *l.* sterling, the *par* being 33*s.* 4*d.* *flemish per pound sterling.*

Note,

16 pennings—1 stiver.
20 stivers—1 guilder or florin.

And in Germany,

12 phennings—1 shilling lub.
16 lubish shillings—1 mark.

8 pennings—1 grote or penny.

12 grotes or pence—1 skilling.

20 skilling—1 pound.

6 phennings—1 grote flem.

6 lubish shill.—1 skill. flem.

7½ marks lub.—1 pound flem.

E X A M P L E S.

1. To how much flemish will 700*l.* sterling amount, exchange at 34*s.* flem. *per l.* ster? — — — — — *Ans.* 1190*l.*

2. To how much sterling will 1190*l.* flemish amount, exchange at 34*s.* *per l.* ster? — — — — — *Ans.* 700*l.*

3. How much flemish must be given for 314*l.* 5*s.* sterling, exchange at 33*s.* 8*d.* flem. *per l.* ster? — — — — — *Ans.* 528*l.* 19*s.* 9*d.*

4. How much sterling must be given for 528*l.* 19*s.* 9*d.* flem. exchange at 33*s.* 8*d.* *per l.* ster? — — — — — *Ans.* 314*l.* 5*s.*

5. How many guilders may I have for 173*l.* 14*s.* 2*d.* ster. exchange at 35*s.* 3½*d.* flem. *per l.* ster? — — — — — *Ans.* 1839 *gu.* 2*st.* 11½ *pen.*

6. How much sterling must I have for 2714 *guil.* 15*st.* exchange at 35*s.* 6*d.* flem. *per l.* ster? — — — — — *Ans.* 254*l.* 18*s.* 1*d.* 1⅙*q.*

7. What quantity of flemish currency must I have for 290*l.* 11*s.* 10*d.* ster. exchange 33*s.* 10*d.* flem. *per l.* ster. and agio at 4½ *per cent*? — — — — — *Ans.* 513*l.* 14*s.* 1*d.* 1⅞*q.*

8. How

8. How much sterling must I receive for 805*l.* 15*s.* Flem. currency, the agio being 4 *per cent.* and exchange 34*s.* 6*d.* Flem. *per l.* ster? — *Ans.* 449*l.* 2*s.* 8*d.* 2 $\frac{1}{2}$ $\frac{7}{8}$ $\frac{9}{16}$ *q.*

9. To how much ster. will 7310 marks, 8*sb.* 9*ph.* amount, exchange at 36*s.* 4*d.* Flem. *per l.* ster?

Ans. 536*l.* 11*s.* 3 $\frac{8}{16}$ $\frac{9}{16}$ *q.*

10. How many marks must be received for 536*l.* ster. exchange at 36*s.* 4*d.* Flem. *per l.* ster? — *Ans.* 7303 marks.

III. With FRANCE.

At France accounts are kept in livres, sols, and deniers; exchange being made by the french crown, whose *par* is 4*s.* 6*d.* sterling.

Note, 12 deniers—a sol or fou.

20 sols—a livre.

3 livres—a crown or ecu.

EXAMPLES.

1. How many livres, &c. will 275*l.* 12*s.* 8*d.* amount to, exchange at 50*d.* *per* ecu? — *Ans.* 396*li* 2*sol.* 4 $\frac{1}{2}$ *d.*

2. To how much sterling will 900 crowns amount, exchange at 52*d.* *per* ecu? — — *Ans.* 105*l.*

3. How many french crowns may I have for 87*l.* 11*s.* 6*d.* ster. exchange at 56 $\frac{1}{2}$ *d.* *per* crown? — *Ans.* 372.

4. To how much ster. will 372 french crowns amount, exchange at 56 $\frac{1}{2}$ *d.* each? — — *Ans.* 87*l.* 11*s.* 6*d.*

IV. With SPAIN, &c.

In Spain they keep their accounts in piastres, rials, and marvadies; reckoning 372 marvadies to a rial, and 8 rials to a piastre, by which they exchange, and whose *par* is 4*s.* 6*d.* sterling.

Note, In Genoa and Leghorn they keep their Accounts in livres, so's, and deniers, as in France, but exchange by the piastre as in Spain, which in Genoa is accounted 5 livres, and at Leghorn 6. At Venice, too, accounts are by some kept in the same manner, and by others in ducats and gros, reckoning 24 gros to a ducat, upon which they exchange, and whose *par* is accounted 4*s.* 4*d.* sterling.

EXAMPLES.

1. How many piastres, &c. shall I receive in Spain for 5*mol.* ster. exchange 50*d.* ster. *per* piastre?

Ans. 2448 piastres.

2. Spain draws upon London for 2448 piaſtres, exchange at 50*d.* *per* piaſtre; how much ſterling will the draught amount to? — — — *Anſ.* 51*ol.*

3. How many livres, &c. muſt be given at Genoa for 175*l.* 15*s.* ſter. exchange at 52*d.* ſter. *per* piaſtre?

Anſ. 4055 *liv.* 15*fol.* 4 $\frac{8}{11}$ *dens.*

4. Genoa draws upon London for 3000 livres: how much ſter. will ſatisfy this draught, exchange at 50 $\frac{1}{2}$ *d.* *per* piaſtre? — — — *Anſ.* 126*l.* 5*s.*

5. How many livres, &c. muſt be received at Leghorn for 705*l.* 16*s.* 4*d.* ſter. exchange at 51 $\frac{1}{2}$ *d.* ſter. *per* piaſtre? — — — *Anſ.* 19735 *liv.* 9*fol.* 1 $\frac{5}{8}$ *den.*

6. Leghorn draws upon London for 12000 *liv.* 14*fol.* exchange at 50*d.* ſter. *per* piaſtre: how much muſt be paid at London for this draught?

Anſ. 416*l.* 13*s.* 9*d.* 3 $\frac{1}{4}$ *q.*

7. How many ducats at Venice will a draught of 427*l.* ſter. amount to, exchange at 49*d.* ſter. *per* ducat?

Anſ. 2091 *duc.* 10 $\frac{2}{7}$ *gro.*

8. Venice draws upon London for 2091 *duc.* 10 *gro.* exchange at 49*d.* ſter. *per* ducat: to how much ſter. does it amount? — — — *Anſ.* 426*l.* 19*s.* 11*d.* 1 $\frac{3}{4}$ *q.*

V. With PORTUGAL.

In Portugal accounts are kept in milreas and reas, reckoning 1000 reas to a milrea, as its name imports; and they exchange by the milrea, whoſe *far* is about 6*s.* 8 $\frac{1}{2}$ *d.* or 6*s.* 9*d.* ſterling.

E X A M P L E S.

1. To how many milreas will 715*l.* amount, exchange at 5*s.* 8*d.* *per* milrea? — — *Anſ.* 2523 *mil.* 529 $\frac{7}{11}$ *reas.*

2. To how many *l.* &c. will a draught of 2523 *milr.* 529 *re.* amount, exchange at 5*s.* 8*d.* *per* milrea?

Anſ. 714*l.* 19*s.* 11*d.* 3 $\frac{1}{11}$ $\frac{7}{11}$ *q.*

3. How many milreas muſt be given for 213*l.* 7*s.* 10*d.* exchange at 5*s.* 9 $\frac{1}{2}$ *d.* *per* milrea?

Anſ. 736 *milr.* 892 $\frac{1}{11}$ $\frac{2}{11}$ *reas.*

4. To how much ſterling will 736 milreas amount, exchange at 5*s.* 9 $\frac{1}{2}$ *d.* *per* milrea? — — *Anſ.* 213*l.* 2*s.* 8*d.*

ALLIGATION.

ALLIGATION is the method of mixing together several simples of different qualities, so that the composition may be of a middle quality: and is commonly distinguished into two principal cases, denominated alligation medial and allegation alternate.

CASE I. ALLIGATION MEDIAL.

Alligation medial is the method of finding the rate of the compound, from having the rates and quantities of the several simples given.

Note, That by the rates are meant the numbers which determine, or define the proportions of the qualities of the simples and compound; such as the given prices of their inegers, or numbers expressing their degrees of fineness, or any other numbers proportional to them. And if any one of the simples be of little or no value with respect to the rest, its rate is supposed to be 0; as water mixed with wine, or alloy with gold and silver.

RULE.

Multiply each quantity by its rate; then divide the sum of the products, by the sum of the quantities, or the whole composition; and the quotient will be the rate of the compound required.

EXAMPLES.

1. A composition being made of 5 lb. of tea, at 7s. per lb. 9 lb. at 8s. 6d. per lb. and $14\frac{1}{2}$ lb. at 5s. 10d. per lb. what is a lb. of it worth? — *Ans.* 6s. 10d. $2\frac{1}{3}\frac{4}{7}q.$

2. What is a gallon of a composition of wine worth, which is made by mixing 4 gallons at 4s. 10d. per gallon, with 7 gallons at 5s. 3d. and $9\frac{3}{4}$ gallons at 5s. 8d. per gallon? — — — *Ans.* 5s. 4d. $1\frac{4}{8}\frac{5}{11}q.$

3. Having mixed together 17 gallons of ale, at 9d. per gallon, 14 at $7\frac{1}{2}d.$ 5 at $9\frac{1}{2}d.$ and 21 at $4\frac{1}{2}d.$ how much per gallon is the mixture worth? — — — *Ans.* $7\frac{1}{3}\frac{1}{7}d.$

4. A mixture being made of 12 bushels of oats, at 1s. 4d. per bush. 9 bushels of peas, at 1s. 7d. and 4 bush. 2 pecks of beans, at 1s. 2d. per bush. what will it be worth per bushel? — — — *Ans.* 1s. 4d. $2\frac{1}{3}\frac{4}{7}q.$

5. A composition being made by mixing 8 gallons of wine, worth 5s. 9d. per gall. with 7 gall. worth 5s. 11d. and 2 gall. of water: what is a gallon of it worth?

Ans. 5s. 8d. $2\frac{1}{4}$ q.

6. Having melted together 7 oz. of gold of 22 carats fine, $12\frac{1}{2}$ oz. of 21 carats fine, and 17 oz. of 19 carats fine: I would know the fineness of the composition?

Ans. $20\frac{1}{3}$ carats fine.

7. Of what fineness is that composition, which is made by mixing 3 lb. of silver of 9 oz. fine, with 5 lb. 8 oz. of 10 oz. fine, and 1 lb. 10 oz. of alloy?—*Ans.* $8\frac{1}{8}$ oz. fine.

CASE II. ALLIGATION ALTERNATE.

Alligation alternate is the method of finding what quantity of each of the simples whose rates are given, will compose a mixture of a given rate; so that it is the reverse of alligation medial, and may therefore be proved by it.

R U L E.

1. Write the rates of the simples in a column under each other.

2. Connect or link with a continued line, the rate of each simple which is less than that of the compound, with one or any number of those which are greater than the compound, and each greater rate with one or any number of the less.

3. Write the difference between the mixture rate and that of each of the simples opposite the rates with which these are linked.

4. Then if only one difference stand against any rate, it will be the quantity belonging to that rate; but if there be several, their sum will be the quantity.

Note. It appears from the rule that many of the questions of this case will admit of various answers each; but from an algebraic process it appears that they will all have infinite varieties of answers, nay, if the expression may be allowed, that they will admit of infinite varieties of infinite varieties of answers. After one or more answers are found by the rule, as many more may be found as you please by increasing or decreasing the quantities in any proportion, or by only increasing or decreasing any one or more single pairs of yoke fellows in any proportion, and leaving the other rates as they are: but as that answer is commonly desired which gives

gives the rates in the least integer numbers, and those the answer to each other, I have to each of the following questions put down such answers as I found by linking the rates together the most possible, and then, where no limitation was proposed, dividing the resulting quantities by their greatest common measure.

EXAMPLES.

1. How much wine at 6s. per gallon, and at 4s. per gallon, must be mixed together, that the composition may be worth 5s. per gallon?—*Ans.* 1 qrt. or 1 gal. or any one equal quantity of each sort.

2. How much sugar at 4d. at 6d. and at 11d. per pound, must be mixed together, that the composition may be worth 7d. per pound.—*Ans.* 1 lb. or 1 stone, or 1 cwt. or any other equal quantity of each sort.

3. How much corn at 2s. 6d. at 3s. 8d. at 4s. and at 4s. 8d. per bushel, must be mixed together, that the compound may be worth 3s. 10d. per bushel?—*Ans.* 2 at 2s. 6d. 2 at 3s. 8d. 3 at 4s. and 3 at 4s. 8d.

4. A composition whose rate may be 7s. 6d. being to be made by mixing together five simples whose rates are 4s. 5s. 8d. 6s. 7s. 4d. and 8s. how much of each must be used? — *Ans.* An equal quantity of the first four sorts, and 14 times the same quantity of the last sort.

5. To mix gold of 19 caracts fine, with gold of 23, of 21, of 18, and of 17 caracts fine, that the compound may be 20 caracts fine: what quantity must be taken of each?

Ans. 2 at $\begin{Bmatrix} 17 \\ 18 \\ 19 \end{Bmatrix}$ and 3 at $\begin{Bmatrix} 21 \\ 23 \end{Bmatrix}$

6. What are the proportions of the quantities of alloy, and gold of 22 caracts fine, which, when mixed together, will make the composition of 20 caracts fine?—*Ans.* There must be 10 times as much gold as there is alloy.

Note, Sometimes one or more of the ingredients, and sometimes the whole composition is limited to a certain quantity; which I divide into the three following cases, or limitations.

LIMITATION I.

When the whole composition is limited to a certain quantity, and that quantity is not found from the method of linking and taking the differences, then you may augment

augment or diminish the quantity of each ingredient in the same proportion as the given quantity is greater or less than the total quantity found from the linking, by saying, As the total quantity so found, is to the given quantity, so is the quantity of each ingredient, found by linking, to the required quantity of each.

EXAMPLES.

1. How much wine at 4s. at 5s. at 5s. 6d. and at 6s. a gallon, must be mixed together, to form a composition of 18 gallons, worth 5s. 4d. a gallon?

Ans. 3 gal. at $\begin{cases} 4s. \\ 5s. \end{cases}$ and 6 gal. at $\begin{cases} 5s. 6d. \\ 6s. \end{cases}$

2. How much gold of 15, of 17, of 18, and of 22 carats fine, must be mixed together, to form a composition of 40 ounces, of 20 carats fine?—*Ans.* 5 oz. of 15, of 17, and of 18, and 25 oz. of 22 carats fine.

N. B. To this case belongs the question concerning king Hiero's crown, which the workman had debased with silver, or copper; and to find what quantity of gold and copper was in it, the famous Archimedes is said to have made two other crowns of the same weight with the former, the one of gold, and the other of silver, or copper; and by putting each into a vessel full of water, the quantity of water expelled by them determined their specific bulks: from which and their given weight it is easier to determine the quantities of gold and copper in the crown by this case of alligation, than by an algebraic process. I shall assume the same numbers which Ronayne has in his algebra, thus,

Suppose the weight of each crown to be 10 lb. and that the water expelled by the copper or silver was 92 lb. by the gold 52 lb. and by the compound crown was 64 lb. that is, their specific bulks were as 92, 52, and 64.

Here then the rates of the simples are 92 and 52, and of the compound 64; whence

64 $\begin{vmatrix} 92 \\ 52 \end{vmatrix}$ $\begin{matrix} 12 \text{ of copper.} \\ 28 \text{ of gold.} \end{matrix}$ $\begin{cases} \text{The sum of these is } 12 + 28 = 40, \text{ which should} \\ \text{have been but 10, wherefore by our rule} \end{cases}$

40 : 10 :: $\begin{cases} 12 : 3 \text{ lb. of copper.} \\ 28 : 7 \text{ lb. of gold.} \end{cases}$

LIMITATION II.

When one of the ingredients is limited to a certain quantity, and that quantity is not found by the method of linking; you may either augment, or diminish, the quantities of all the rest in the same proportion as the given quantity

quantity is greater or less than the quantity of the limited simple found by linking, by stating as in the first limitation : * Or, you may only augment, or diminish, in the above proportion, that part of the quantities of the ingredients with which the limited one is linked, which is the difference of the mixture rate and the rate of the limited simple, and add the resulting quantity to the other parts, instead of the said difference; keeping the quantities of the other simples unaltered.

EXAMPLES.

1 How much wine at 5s. at 5s. 6d. and at 6s. the gallon, must be mixed with 3 gallons at 4s. the gallon, that the mixture may be worth 5s. 4d. a gallon?

Ans. $\left\{ \begin{array}{l} 3 \text{ gal. at } 5s. \\ 6 \text{ - - - } 5s. \text{ 6d.} \\ 6 \text{ - - - } 6s. \end{array} \right\}$ by proportioning all the quantities :

Or, $\left\{ \begin{array}{l} 10 \text{ gal. at } 5s. \\ 8\frac{4}{5} \text{ - - - } 5s. \text{ 6d.} \\ 8\frac{4}{5} \text{ - - - } 6s. \end{array} \right\}$ by proportioning only the difference of the mixture and limited rates.

2. How much gold of 15, of 17, and of 22 caracts fine, must be mixed with 5 ounces of 18 caracts fine, that the composition may be of 20 caracts fine?

Ans. $\left\{ \begin{array}{l} 5 \text{ oz. of } 15 \text{ caracts fine} \\ 5 \text{ - - - } 17 \text{ - - - - -} \\ 25 \text{ - - - } 22 \text{ - - - - -} \end{array} \right\}$ by proportioning all the quantities.

Or, $\left\{ \begin{array}{l} 2 \text{ oz. of } 15 \text{ caracts fine} \\ 2 \text{ - - - } 17 \text{ - - - - -} \\ 13 \text{ - - - } 22 \text{ - - - - -} \end{array} \right\}$ by proportioning only the difference of the mixture and limited rates.

L R.

* Hence we may observe that Mr. Malcolm has inadvertently given a rule in pag. 569 of his arithmetic, for questions of this sort when the limited simple is only once linked, which will not always give true answers; he says, " If the simple whose quantity is limited is only once linked, we need do no more than raise or diminish the quantity of that one simple with which it is link'd, and leave the rest as they are." Instead of which, if he had considered that the simple with which the limited one is linked, may also be linked with some one or more of the rest, I apprehend he would have said, Raise or diminish that part of the simple with which the limited one is linked, which is the difference between the mixture rate and the rate of the limited simple.

LIMITATION. III.

If more than one of the simples be limited, find by *case I*, what will be the rate of a mixture made of the given quantities of the limited simples only; then consider this as the rate of a limited simple; whose quantity is the sum of the first given limited simples, from which and the rates of the limited simples, by the second limitation, calculate the quantity of each.

EXAMPLES.

1. How much wine at 5s. 6d. and at 6s. a gallon, must be mixed with 3 gallons at 4s. and 3 gallons at 5s. a gallon, that the mixture may be worth 5s. 4d. a gallon?

Ans. 6 gal. at 5s. 6d. and 6 gal. at 6s. a gallon.

2. How much gold of 15 and of 17 caracts fine, must be mixed with 5 ounces of 18, and 13 ounces of 22 caracts fine, that the composition may be of 20 caracts fine?

Ans. 2 oz. of each sort.

I N V O L U T I O N.

A Power is a number produced by multiplying any given number continually into itself a certain number of times.

Any number is called the first power of itself; if it be multiplied by itself, the product is called the second power, and sometimes the square; if this be multiplied by the first power again, the product is called the third power, and sometimes the cube; and if this be multiplied by the first power again, the product is called the fourth power, &c. that is, the power is denominated from the number which exceeds the multiplications by 1.

Thus: 3 is the first power of 3.

$3 \times 3 = 9$ is the second power of 3.

$3 \times 3 \times 3 = 27$ is the third power of 3.

$3 \times 3 \times 3 \times 3 = 81$ is the fourth power of 3.

&c.

&c.

And in this manner may be calculated the following

TA-

TABLE of the first twelve Powers of the nine digits.

1st power	2	3	4	5	6	7	8	9
2d power	4	9	16	25	36	49	64	81
3d power	8	27	64	125	216	343	512	729
4th power	16	81	256	625	1296	2041	4096	6561
5th power	32	243	1024	3125	7776	16807	32768	59049
6th power	64	729	4096	15625	46656	117649	262144	531441
7th power	128	2187	16384	78125	279936	823543	2097152	4782969
8th power	256	6561	65536	390625	1679616	5764801	16777216	43046721
9th power	512	19683	262144	1953125	10077696	40353607	134217728	387420489
10th power	1024	59049	1048576	9765625	60466176	282475249	1073741824	3486784401
11th power	2048	177147	4194304	48828125	362797056	1977326743	8589934592	31381059809
12th power	4096	531441	16777216	244140625	2176782336	13841287201	68710476736	282429536481

Note;

NOTE 1. The number which exceeds the multiplications by 1 is called the index, or exponent of the power: so the index of the first power is 1, that of the second power is 2, and that of the third is 3, &c.

2. Powers are commonly denoted by writing their indices above the first power: so the second power of 3 may be denoted thus 3^2 , the third power thus 3^3 , the fourth power thus 3^4 , &c. and if the given number consist of several figures, a crooked line is sometimes drawn between them and the index: thus 5031^6 , denotes the sixth power of 5031.

Involution is the finding of powers; to do which, from their definition, there evidently comes this

R U L E.

Multiply the given number, or first power, continually by itself, till the number of multiplications be 1 less than the index of the power to be found, and the last product will be the power required.

NOTE 1. Whence, because fractions are multiplied by taking the products of their numerators and of their denominators, they will be involved by raising each of their terms to the power required.—And if a mixt number be proposed, either reduce it to an improper fraction, or reduce the vulgar fraction to a decimal, and proceed by the m^o.

2. The raising of powers will be a little shortened by working according to this observation; viz. whatever two or more powers are multiplied together, their product is the power whose index is the sum of the indices of the factors; or if a power be multiplied by itself, the product will be the power whose index is double of that which is multiplied: so, if I would find the sixth power; I might multiply the given number twice by itself for the third power, then the third power into itself would give the sixth power: or if I would find the seventh power; I might first find the third and fourth, and their product would be the seventh: or lastly, if I would find the eighth power; I might first find the second, then the second into itself would be the fourth, and this into itself would give the eighth.

E X A M P L E S.

1. What is the second power of 45? — *Ans.* 2025.
2. What is the second power of 416? — *Ans.* 173056.
3. What is the second power of .027? — *Ans.* .000729.
4. What is the third power of 3.5? — *Ans.* 42.875.
5. What is the fourth power of 71.8?

Ans. 26576499.4576.

6. What is the fifth power of .029?

Ans. .000000020511149.

7. What

7. What is the sixth power of 5.03?

Ans. 16196.005304479729.

8. What is the second power of $\frac{3}{4}$? — *Ans.* $\frac{9}{16}$.

9. What is the third power of $\frac{5}{9}$? — *Ans.* $\frac{125}{729}$.

10. What is the second power of $3\frac{1}{3}$?

Ans. $12\frac{8}{9}$ or 11.56.

E V O L U T I O N.

THE root of any given number, or power, is such a number, as being multiplied into itself a certain number of times, will produce the power; and is denominated the first, second, third, fourth, &c. root, respectively as the number of multiplications made of it to produce the given power is 0, 1, 2, 3, &c. that is, the name of the root is taken from the number which exceeds the multiplications by 1, like the name of the power in involution.

NOTE 1. The index of the root, like that of the power in involution, is 1 more than the number of the multiplications necessary to produce the power or given number.

2. Roots are sometimes denoted by writing $\sqrt{}$ before the power, with the index of the root against it: so the third root of 50 is $\sqrt[3]{50}$, and the second root of it is $\sqrt{50}$, the index 2, being omitted; which index is always understood when a root is named or wrote without one. But if the power be expressed by several numbers with the sign +, or —, &c. between them, then a line is drawn from the top of the sign of the root, or radical sign, over all the parts of it: so the third root of $47-15$ is $\sqrt[3]{47-15}$. And sometimes roots are designed like powers, with the reciprocal of the index of the root above the given number, and a line between them if the number have more than one figure in it.

So the root of 3 is $3^{\frac{1}{2}}$, the root of 50 is $50^{\frac{1}{2}}$, and the third root of it is $(50^{\frac{1}{2}})^{\frac{1}{3}}$;

also the third root of $47-15$ is $(47-15)^{\frac{1}{3}}$. And this method of notation has justly prevailed in the modern algebra; because such roots, being considered as fractional powers, need no other directions for any operations to be made with them than those for integral powers.

3. A number is called a complete power of any kind when its root of the same kind can be accurately extracted; but if not, the number is called an imperfect power, and its root a surd or irrational quantity: so 4 is a complete power of the second kind, its root being two; but an imperfect power of the third kind, its third root being a surd quantity.

Evolution is the finding of the roots of numbers, either accurately, or in decimals till the error be less than any proposed number.

The power is first to be prepared for extraction, or evolution, by dividing it, from the place of units, to the left in integers and to the right in decimal fractions, into periods containing each so many places of figures as are denominated by the index of the root, if the power contain a complete number of such periods: if it do not, the defect will be either on the right hand, or left, or both; if the defect be on the right hand, it may be supplied by annexing ciphers, and after this whole periods of ciphers may be annexed to continue the extraction with, if necessary; but if there be a defect on the left, such defective period must remain unaltered, and is accounted the first period of the given number, just the same as if it were complete.

Now this division may be conveniently made by writing a point over the place of units, and also over the last figure of every period on both sides of it; that is, over every second figure if it be the second root, over every third if it be the third root, &c. Thus, to point this number

21035896·12735 for the second root it will be 2103589
6·127350; but for the third root, thus 21035896·127350;
and for the fourth, thus 21035896·12735000.

Note. The root will contain just so many places of figures as there are periods or points in the given power; and they will be integers, or decimals, respectively as the periods are so from which they are found, or to which they correspond; that is, there will be so many integer figures in the root, as there are periods of integers in the given number.

RULE for extracting the square or second root of integers, or decimals, or both mixed together.

1. Find, from the table of powers in page 107, or otherwise, a square number either equal to, or the next less than the first period; which subtract from it, and place the

the root of the square on the right of the given number, after the manner of a quotient in division, for the first figure of the root required.

2. To the remainder annex the second period for a dividend; and on the left thereof write the double of the root already found, after the manner of a divisor.

3. Consider what figure, which, if annexed to the divisor, and the result multiplied by it, the product may be equal to, or the next less than the dividend, and it will be the next figure of the root.

4. From the dividend subtract the product, and to the remainder bring down the next period, for a new dividend: to which, as before, find a divisor by doubling the figures already found in the root; and from these find the next figure of the root, as in the last article; and continue the operation still in the same manner till all the periods be used.

Note 1. In the last place of the following answers, I write such figure as is nearest the truth, whether it be too great or too little, i. e. if the next figure would equal or exceed 5, I increase the last place by 1, if not, I do not alter it: also when the root is too great, — is put after it, but when too little, +.

2. When the root is to be extracted to a great number of places, the work may be much abbreviated thus: having proceeded in the extraction after the common method till you have found one more than half the required number of figures in the root, the rest may be found by dividing the last remainder by its corresponding divisor, annexing a cipher to every dividend, as in division of decimals; or rather, without annexing ciphers, by omitting continually the right hand figure of the divisor, after the manner of the third contraction in division of decimals in page 59. So the operation for the root of 2 to 12 or 13 places, may be thus.

E V O L U T I O N.

2)1'41421356237† root.

I

$$\begin{array}{r|l} 24 & 100 \\ 4 & 96 \end{array}$$

$$\begin{array}{r|l} 281 & 400 \\ 1 & 281 \end{array}$$

$$\begin{array}{r|l} 2824 & 11900 \\ 4 & 11296 \end{array}$$

$$\begin{array}{r|l} 28282 & 60400 \\ 2 & 56564 \end{array}$$

$$\begin{array}{r|l} 282841 & 383600 \\ 1 & 282841 \end{array}$$

$$\begin{array}{r|l} 2828423 & 10075900 \\ 3 & 8485269 \end{array}$$

$$2828425) 1590631 (56237 \dagger$$

$$\begin{array}{r} 176418 \\ \hline \end{array}$$

$$\begin{array}{r} 6712 \\ \hline \end{array}$$

$$\begin{array}{r} 1055 \\ \hline \end{array}$$

$$\begin{array}{r} 206 \\ \hline \end{array}$$

$$\begin{array}{r} 8 \\ \hline \end{array}$$

E X A M P L E S.

1. What is the root of 2025? — *Ans.* 45.
2. What is the root of 17'3056? — *Ans.* 4'16.
3. What is the root of '000729? — *Ans.* '027.
4. What is the root of 3? — *Ans.* 1'73205†.
5. What is the root of 5? — *Ans.* 2'236068—.
6. What is the root of 6? — *Ans.* 2'44949—.
7. What

7. What is the root of 7? — *Ans.* 2.645751+.
8. What is the root of 10? — *Ans.* 3.162278—.
9. What is the root of 11? — *Ans.* 3.316625—.

RULES for the square roots of vulgar fractions and mixt numbers.

First prepare all vulgar fractions by reducing them to their least terms, both for this and all other roots. Then

1. Take the root of the numerator and of the denominator for the respective terms of the root required. And this is the best way if the denominator be a complete power. But if it be not,

2. Multiply the numerator and denominator together; take the root of the product; this root being made the numerator to the denominator of the given fraction, or made the denominator to the numerator of it, will form the fractional root required.

That is, $\sqrt{\frac{a}{b}} = \frac{\sqrt{ab}}{\sqrt{b}} = \frac{b}{\sqrt{ab}}$. And this rule will serve whether the root be finite or infinite. Or,

3. Reduce the vulgar fraction to a decimal, and extract its root.

4. Mixt numbers may be either reduced to improper fractions, and extracted by the first and second rule; or, the vulgar fraction may be reduced to a decimal, then joined to the integer, and the root of the whole extracted.

E X A M P L E S.

1. What is the root of $\frac{25}{36}$? — — *Ans.* $\frac{5}{6}$.
2. What is the root of $\frac{27}{121}$? — — *Ans.* $\frac{3}{11}$.
3. What is the root of $\frac{9}{121}$? — *Ans.* .866095+.
4. What is the root of $\frac{5}{121}$? — *Ans.* .645497+.
5. What is the root of $17\frac{1}{8}$? — *Ans.* 4.168333+.

GENERAL RULE for the extraction of all roots of integers, or decimals.

1. Consider what figure, to which if ciphers be annexed, it will be nearer the root of the given number than any other single figure with ciphers: which figure will be the root of the next less power to the first period, of the same height with the given power, if the second figure of the root be less than 5; but if it exceed or be only equal to 5,

provided any more figures be to follow it, then the nearest figure will be the root of the next greater power than the first period. And in most cases it is very evident whether the second place of the root be less than 5 or not: for if the next less power be much nearer the first period than the next greater, the second figure is less than 5, and the contrary; but if the one power be almost as near to the first period as the other, the thing is not always so evident; for though the second place be always equal to or greater than 5 when the next greater power is any thing, though ever so little nearer to the first period than the next less power, yet it is not always less than 5 when the next less power is nearer than the next greater: so that when this is the case, annex 5 to the root of the next less power, and raise the result to the height of the given power, which will shew whether 5 be too great or too little.

2. Having found the nearest figure, involve it to the height of the given power; then take the difference between this power and the first period, and annex a cipher to it for a dividend.

3. And for a divisor, multiply the index of the root by such a power of the first found nearest figure whose index is less by 1 than that of the given power, or required root.

4. Their quotient will be the next figure of the root if the first figure be less than just; but if the first figure be greater than the root, annex a 0 to it, and subtract the quotient figure from the result, and consider the remainder as the two first figures of the root.

5. Involve the part of the root already found to the same height with the given power, and annex a 0 to the difference between this power and the first periods of the given number, for a dividend, as before. Then, as before also, find a divisor, by multiplying the index by such a power of the figures already found of the root whose index is less by 1 than that of the given power: let the two figures of the root, with a 0 annexed, be augmented or diminished by the quotient of these, according as they are too little or too great; and in making the addition or subtraction, let the units place of the quotient correspond with, or be placed under, the cipher annexed to the former

mer figures. Then, from the sum or difference, find more figures of the root in the same manner: and so on as far as you please.

Note 1. Generally, the first division may be continued to one place or figure, the second to two, the third to four, and so on, always doubling the number of figures already found, by annexing a cipher continually to the remainders, or by omitting successively the right-hand figures of the divisor after the manner of the third contraction in division of decimals: and by this method the successive sums or differences are commonly nearer to the true root than any other expression of the same number of figures; so that when they are too little, the last figure increased by 1, makes them too great; and when they are in excess, the last figure diminished by 1, makes them defective. Though very often the division may be continued to many more figures of the root than these; and there is no danger in taking more figures; for if they happen to be wrong, the next division will correct them. It is to be noted also, that the several divisors and dividends are to be accounted integers.

2. The extraction of roots is greatly expedited by observing what integer numbers multiplied together produce the index of the required root. and making such extractions as are denominated by these numbers. Thus, instead of the fourth root, extract twice the square root; instead of the sixth, extract first the square root, and then the cube root of that; instead of the eighth, extract thrice the square root; and instead of the ninth, extract twice the cube root; and so on.

EXAMPLES.

1. To find the cube root of 2103589612735?

	2103589612735	30 —2
$3^3 = 27$		280
$3^2 \times 3 = 27$	60(2	—39
$28^3 = 21952$		27610
		—50464 &c.
$28^2 \times 3 = 2352$	9170 (39	27604953 &c.
		—39188
$2761^3 = 21047437081$		2760495260812
$2761^2 \times 3 = 22869363$	115409540(50464 &c.	
$27604953^3 = 21035897023228$	&c.	
$27604953^2 \times 3 = 228610$	895878 &c.(39188 &c.	

Note,

Note, It appears from this example, that we need find but a few of the first figures of the divisors and dividends, which greatly facilitates the work, for then the powers may be raised after the manner of the second contraction in multiplication of decimals.

2. To extract the fourth root of 21035896.12735.

$$\begin{array}{r}
 \begin{array}{cccccccc}
 \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\
 21035896.12735(4586.49061(67.7236340 \text{ root.} \\
 16 & & & & 36 & & &
 \end{array} \\
 \hline
 \begin{array}{r|l}
 85 & 503 \\
 5 & 425 \\
 \hline
 908 & 7858 \\
 8 & 7264 \\
 \hline
 9166 & 59496 \\
 6 & 54996 \\
 \hline
 91724 & 450012 \\
 4 & 366896 \\
 \hline
 \end{array}
 \end{array}$$

$$\begin{array}{r}
 \begin{array}{r|l}
 127 & 986 \\
 7 & 889 \\
 \hline
 1347 & 9749 \\
 7 & 9429 \\
 \hline
 13542 & 32006 \\
 2 & 27084 \\
 \hline
 135443 & 492210 \\
 3 & 406329 \\
 \hline
 \end{array}
 \end{array}$$

83116(9061

85881(6340

564

4615

14

552

10

Note, Here I make two extractions of the square root, according to what was observed in note 1. page 115: for $2 \times 2 = 4$, 2 being the index of the square root, and 4 that of the root required.

3. To find the fifth root of 21035896.12735.

	21035896.12735	3
$3^5 =$	243	—08
$3^4 \times 5 = 405$. 330 (08	292
		—53
$292^5 =$	2122826 &c.	29147
		—1029
$292^4 \times 5 = 3634$ &c.	.. 19237 (53	
		291468971 the root.
$29147^5 =$	21036267370 &c.	
$29147^4 \times 5 = 360865$ &c. 371243 (10 29	

Note, In the first division of this example, the first figure of the quotient being a cipher, it was necessary to continue it to another place, to make an alteration in the first figure of the root.

4. What is the third root of 2? — *Ans.* 1.259921+.
5. What is the fourth root of 2? — *Ans.* 1.189207+.
6. What is the fifth root of 2? — *Ans.* 1.148699—.
7. What is the sixth root of 21035896.12735? — *Ans.* 16.61474—.
8. What is the sixth root of 2? — *Ans.* 1.122462—.
9. What is the seventh root of 21035896.12735? — *Ans.* 11.12083+.
10. What is the seventh root of 2? — *Ans.* 1.104089+.
11. What is the eighth root of 21035896.12735? — *Ans.* 8.22943+.
12. What is the eighth root of 2? — *Ans.* 1.090508—.
13. What is the ninth root of 21035896.12735? — *Ans.* 6.51122+.
14. What is the ninth root of 2? — *Ans.* 1.080059+.

GENERAL RULES for extracting any root out of a vulgar fraction, or mixt number.

1. If the given fraction have a finite root of the kind required, it is best to extract the root out of the numerator and denominator, for the terms of the root required.

2. But

2. But if the fraction be not a complete power, it may be thrown into a decimal, and then extracted. Or,

3. Take either of the terms of the given fraction for the corresponding term of the root; and for the other term of the root, extract the required root of the product, arising from the multiplication of such a power of the first assigned term of the root whose index is less by 1 than that of the given power, by the other term of the given number. This rule will do when the root is either finite or infinite.

$$\text{That is } \sqrt[n]{\frac{a}{b}} = \frac{a^{\frac{1}{n}}}{b^{\frac{1}{n}}} = \frac{a}{b a^{1-\frac{1}{n}}}^{\frac{1}{n}}.$$

4. Mixt numbers may be reduced either to improper fractions or decimals, and then extracted.

EXAMPLES.

1. What is the cube root of $\frac{8}{27}$? — *Ans.* $\frac{2}{3}$.
2. What is the fourth root of $\frac{800}{1000}$? — *Ans.* $\frac{2}{5}$.
3. What is the cube root of $\frac{1}{8}$? — *Ans.* $\frac{1}{2}$.
4. What is the cube root of $2\frac{10}{27}$? — *Ans.* $\frac{4}{3}$ or $1\frac{1}{3}$.
5. What is the third root of $7\frac{1}{2}$? — *Ans.* 1.930979 .

Of PROPORTION in GENERAL.

NUMBERS are compared together to discover the relations they have to each other.

There must be two numbers to form a comparison: the number which is compared, being wrote first, is called the antecedent; and that to which it is compared, the consequent.

Numbers are compared to each other two different ways: the one comparison considers the difference of the two numbers, and is called arithmetical relation, the difference being sometimes named the arithmetical ratio; and the other considers their quotient, which is termed geometrical relation, and the quotient the geometrical ratio. So of these numbers 6 and 3; the difference, or arithmetical ratio, is $6-3=3$; and the geometrical ratio is $\frac{6}{3}=2$.

Note,

Note, Ratios are here, always, considered as the result of the greater term of comparison diminished, or divided, by the less; not regarding whether of them be the antecedent.

If two or more couplets of numbers have equal ratios, or differences, the equality is termed proportion; and their terms similarly posited, that is, either all the greater, or all the less taken as antecedents, and the rest as consequents, are called proportionals. So the two couplets 2, 4, and 6, 8, taken thus, 2, 4, 6, 8, or thus, 4, 2, 8, 6, are arithmetical proportionals; and the two couplets 2, 4, and 8, 16, taken thus, 2, 4, 8, 16, or thus, 4, 2, 16, 8, are geometrical proportionals.

To denote numbers as being geometrically proportional, the couplets are separated by a double colon, and a colon is wrote between the terms of each couplet: we may also denote arithmetical proportionals by separating the couplets with a double colon, and writing a colon, turned horizontally, between the terms of each couplet. So the above arithmetics may be wrote thus, $2 \dots 4 :: 6 \dots 8$, and $4 \dots 2 :: 8 \dots 6$; where the first antecedent is less or greater than its consequent, by just so much as the second antecedent is less or greater than its consequent: and the geometricals thus, $2 : 4 :: 8 : 16$, and $4 : 2 :: 16 : 8$; where the first antecedent is contained in or contains its consequent, just so often as the second is contained in or contains its consequent.

Note, It is common to read the geometricals $4 : 2 :: 16 : 8$, thus, 4 is to 2 as 16 to 8.

Proportion is distinguished into continual and discontinual.

If, of several couplets of proportionals, wrote down in a series, the difference or ratio of each consequent and the antecedent of the next following couplet be the same as the common difference or ratio of the couplets, the proportion is said to be continual, and the numbers themselves a series of continual proportionals, or an arithmetical or geometrical progression. So 2, 4, 6, 8 form an arithmetical progression; for $4 - 2 = 2$, $6 - 4 = 2$, $8 - 6 = 2$; and 2, 4, 8, 16 a geometrical progression; for $\frac{4}{2} = \frac{8}{4} = \frac{16}{8} = 2$. But

But if the difference or ratio of the consequent of one couplet and the antecedent of the next couplet, be not the same as the common difference or ratio of the couplets, the proportion is said to be discontinued. So 4, 2, 8, 6 are in discontinued arithmetical proportion; for $4-2=8-6=2$, and $8-2=6$: also 4, 2, 16, 8 are in discontinued geometrical proportion; for $\frac{4}{2}=\frac{16}{8}=2$, and $\frac{16}{2}=8$.

If the succeeding terms of a progression exceed each other, it is called an ascending progression or series; if the contrary, a descending series.

So $\begin{cases} 0, 1, 2, 3, 4, \&c. \text{ is an ascending arithmetical series.} \\ 1, 2, 4, 8, 16, \&c. \text{ is an ascending geometrical series.} \end{cases}$
and $\begin{cases} 4, 3, 2, 1, 0, \&c. \text{ is a descending arithmetical series.} \\ 16, 8, 4, 2, 1, \&c. \text{ is a descending geometrical series.} \end{cases}$

Note, The first and last terms of a progression are called the extremes; and the other terms, the means.

ARITHMETICAL PROGRESSION.

AN arithmetical progression is a series whereof the succeeding terms are either all greater or less than their adjacent preceding terms by the same number or difference.

Note, the fundamental property of an arithmetical progression, from which almost all its other properties are deducible, and which evidently follows from its construction, is, that the sum of any two of its terms is equal to the sum of any other two terms taken at an equal distance, but on contrary sides of the former; or that the double of any one term is equal to the sum of any two terms taken at an equal distance from it on each side. And of any two couplets in discontinued arithmetical proportion, the two sums made by adding the antecedent of each to the consequent of the other are equal.

PROBLEM I.

From any given term, and with any given difference, to construct an arithmetical series.

RULE.

For an ascending series, add the difference to the first term, the sum will be the second; add the difference to
the

the second, the sum will be the third; and so on, continually adding the difference to the term last found for the next succeeding term. And for a descending series, subtract continually the common difference for the several succeeding terms.

EXAMPLE.

It is required to raise an arithmetical ascending, and also a descending series, with the common difference 3, and first term 24.

PROBLEM II.

From any given term of an arithmetical series, whose common difference is known, to find any remote term, whose distance from the given term is known.

RULE.

To or from the given term, add or subtract the product of the common difference multiplied by the distance of the two terms; and the sum or difference will be the term required, according as it is to be greater or less than the given term.

Note, By the distance of any two terms, is meant one less than the whole number of terms, supposed in the series from the one term to the other inclusive — Though the chief use of this problem be when the first or least term is given, to find the greatest, it will serve equally well for any other two terms.

EXAMPLES.

1. What is the m th term of the ascending or descending series, whose common difference is d and first term a ?

Ans. $a \pm d \times m - 1$.

2. What is the 9th term of the ascending series, whose first term is 1 and common difference is 3? — *Ans.* 25.

3. What is the 12th term of the ascending series, whose first term is 0 and common difference 2? — *Ans.* 22.

4. What is the 7th term of the descending series, whose first term is 60 and common difference 4? — *Ans.* 36.

5. What is the 15th term of the descending series, whose first term is 28 and common difference 2? — *Ans.* 0.

6. If 7 be any term of a series, whose common difference is 5; what is the term whose distance from it is expressed by 13, the required term being the greater? — *Ans.* 72.

PROBLEM III.

Between any two given numbers to find any number of arithmetical means.

RULE.

Divide the difference of the two given numbers by one more than the number of means required, and the quotient will be the common difference; then from one of the given extremes and the common difference, find the required means by problem 1.

EXAMPLES.

To find m arithmetical means between a and z .

First $\frac{z - a}{m + 1} =$ the common difference; then the means

are $a + \frac{z - a}{m + 1}$, $a + 2 \frac{z - a}{m + 1}$, $a + 3 \frac{z - a}{m + 1}$, &c. unto

$$a + m \frac{z - a}{m + 1}.$$

2. To find an arithmetical mean between 2 and 6.

Ans. 4.

3. To find two arithmetical means between 1 and 10.

Ans. 4 and 7.

4. To find three arithmetical means between 0 and 24.

Ans. 6, 12, and 18.

5. To find nine arithmetical means between 2 and 10.

Ans. $2\frac{4}{5}$, $3\frac{1}{5}$, $4\frac{2}{5}$, $5\frac{3}{5}$, 6, $6\frac{4}{5}$, $7\frac{1}{5}$, $8\frac{2}{5}$ and $9\frac{3}{5}$.

PROBLEM IV.

Two terms, with their distance in the series being given; to find any other term whose distance from either of the former is given.

RULE.

Divide the difference of the given terms by their distance, and the quotient will be the common difference of the series; then find the terms required by prob. 2.

Note 1. When, in the following examples, the distance of the given terms is not expressed, they are supposed to be adjacent terms. Also when the place of the required term is given, but not the places of the given terms, they are supposed

posed to be the two first terms of the series. And, in both these cases, the difference of the given terms is the common difference of the series.

2. This problem is adapted for finding a mean proportional between, or a third proportional to any two given numbers, as well as for finding any other term.

EXAMPLES.

1. Given the distance n of the two terms a and z ; it is required to find a term, whose distance from a is denoted by m .

First $\frac{a \oslash z}{n}$ = the common difference; then $a + \frac{a \oslash z}{n} \times m$ = the term required.

Or, If the question be proposed in this manner: Given the m th term a , and the n th term z , of a series; to find the p th term.

Then because $m \oslash n$ is the distance between a and z , we shall have $\frac{a \oslash z}{m \oslash n}$ = the common difference; and since $m \oslash p$ is the distance between a and the required term, we shall have $a + \frac{a \oslash z}{m \oslash n} \times m \oslash p$ = the term required.

2. What is the third term of the arithmetical series, whose first and second terms are 3 and 7? Or, To find a third arithmetic proportional to the numbers 3 and 7.

Ans. 11.

3. What is the second term of the series, whose first and third terms are 3 and 11? Or, To find a mean between the numbers 3 and 11.

Ans. 7.

4. What is the 20th term of the series, whose two first terms are 0 and 5? — — —

Ans. 95.

5. What is the 34th term of the series, whose first term is $5\frac{1}{2}$ and eleventh term is 10? — — —

Ans. $28\frac{1}{2}$.

6. What is the eleventh term of the descending series, whereof $9\frac{1}{2}$ and $5\frac{1}{2}$ are the second and seventh terms?

Ans. 2.

7. Given the distance 12, of the two terms 5 and 19; to find a term whose distance from 5 shall be 40; the required term being greater than 5. — — —

Ans. $51\frac{1}{2}$.

PROBLEM V.

Given one of the extremes, the common difference, and the number of terms of an arithmetical series; to find

1. The other extreme.

RULE.

To or from the given term, according as it is the least or greatest, add or subtract the product of the common difference multiplied into 1 less than the number of terms, and the sum or difference will be the term required.

2. The sum of all the terms of the series.

RULE.

Multiply the sum of the extremes by the number of terms, and half the product will be the sum of the series.

Thus, if a represent the less extreme,

z the greater,

d the common difference,

n the number of terms, and

s the sum of the series;

$$\text{then } \begin{cases} z = a + d \times \overline{n-1.} \\ a = z - d \times \overline{n-1.} \end{cases} \text{ and } \begin{cases} s = \frac{21 + dn - d}{2} \times n, \\ s = \frac{2z - dn + d}{2} \times n. \end{cases}$$

EXAMPLES.

1. Given the least term 3, the common difference 2, and the number of terms 9: to find the greatest term and the sum of the series?—*Ans.* The greatest term is 19, and the sum of the series is 99.

2. If the greatest term be 70, the common difference 3, and the number of terms 21; what is the least term, and the sum of the series?—*Ans.* The least term is 10, and the sum is 840.

3. A debt can be discharged in a year, by paying 1 shilling the first week, 3 shillings the second, and so on, always 2 shillings more every week: what is the debt, and what will the last payment be?—*Ans.* The last payment will be 5*l.* 3*s.* and the debt is 135*l.* 4*s.*

PROBLEM VI.

Given the extremes and the common difference, to find

1. The number of terms.

RULE.

Divide the difference of the extremes by the common difference, add 1 to the quotient, and the sum will be the number of terms.

2. The sum of the series.

Having found the number of terms, the sum of the series will be had by the 2d case of problem 5.

Thus, using the same symbols as before, $n = \frac{z-a}{d} + 1$.

and $s = a + z \times \frac{z-a+d}{2d}$.

EXAMPLES.

1. If the extremes be 3 and 19, and the common difference 2; what is the number of terms, and the sum of the series? — *Ans.* The number of terms is 9, and the sum is 99.

2. If the extremes be 10 and 70, and the common difference 3; what is the number of terms, and the sum of the series? — *Ans.* The number of the terms is 21, and the sum is 840.

3. What debt can be discharged, and in what time, supposing the first week the payment be 1s. and the payments every week following to increase by 2s. till the last payment be 5l. 3s? — *Ans.* The debt is 135l. 4s. and will be discharged in a year or 52 weeks.

PROBLEM VII.

Given the extremes and the number of terms, to find

1. The common difference.

RULE.

This is found as in problem 4, by dividing the difference of the extremes by 1 less than the number of terms.

M 3

2. The

2. The sum of the series.

This is had from the 2d case of problem 5.

Thus, $d = \frac{z-a}{n-1}$, and $s = \frac{a+z}{2} \cdot n$.

EXAMPLES.

1. If the extremes be 3 and 19, and the number of terms 9; what is the common difference and sum of the series?—*Ans.* The difference is 2, and the sum is 99.

2. If the extremes be 10 and 70, and the number of terms 24; what is the common difference, and the sum of the series?—*Ans.* The difference is 3, and the sum is 840.

3. What debt can be discharged in a year, by weekly payments in arithmetical progression, whereof the first term or payment is 1s. and the last is 5l. 3s. and what is the common difference of the series of payments?

Ans. The difference is 2s. and the debt is 135l. 4s.

PROBLEM VIII.

Given one of the extremes, the common difference, and the sum of the series; to find

1. The other extreme.

RULE.

To find the greatest term; add twice the product of the common difference and sum of the series to the square of the difference between the least term and half the common difference; then the square root of the sum diminished by half the common difference will be the greatest term. To find the least term; from the square of the sum of half the common difference and the greatest term, subtract double the product of the common difference and sum of the series; then the square root of the difference added to, or taken from, half the common difference, will give the least term.

2. The number of terms.

Having now both the extremes and the common difference, the number of terms will be found by the first case of problem 6.

thus

$$\text{Thus } \begin{cases} z = a + \frac{1}{2}d^2 + 2ds - \frac{1}{2}d \\ a = \frac{1}{2}d + z + \frac{1}{2}d^2 - 2ds \end{cases}$$

$$\text{and } \begin{cases} n = \frac{\frac{1}{2}d - a + a + \frac{1}{2}d^2 + 2ds}{d} \\ n = \frac{\frac{1}{2}d + z + \frac{1}{2}d^2 - 2ds}{d} \end{cases}$$

EXAMPLES.

1. If the least term be 3, the common difference 2, and the sum of the series 99; what is the greatest term, and the number of the terms?—*Ans.* The greatest term is 19, and the number of terms 9.

2. If the greatest term be 70, the common difference 3, and the sum of the series 840; what is the least term, and the number of the terms?—*Ans.* The least term is 10, and the number of terms 21.

3. In what time will a debt of 135*l.* 4*s.* be discharged by weekly payments in arithmetical progression, the first term or payment being 1*s.* and the common difference 2*s.* and what will the last payment be?—*Ans.* The last payment will be 5*l.* 3*s.* and the debt will be discharged in a year.

PROBLEM IX.

Given one extreme, the sum of the series, and the number of terms; to find

1. The other extreme.

RULE.

Divide the sum of the series by the number of terms; then the double of the quotient diminished by the given extreme, is equal to the term required.

2. The common difference.

Having

Having now both the extremes and the number of terms, the common difference will be found by problem 4, or by the first case of problem 7.

$$\text{Thus } z = \frac{2s}{n} - a, \text{ or } a = \frac{2s}{n} - z; \text{ and } \begin{cases} d = \frac{s - an}{n-1} \times \frac{2}{n} \\ d = \frac{nz - s}{n-1} \times \frac{2}{n} \end{cases}$$

EXAMPLES.

1. If the least term be 3, the number of terms 9, and the sum of the series 99; what is the greatest term, and the common difference?—*Ans.* The greatest term is 19, and the common difference 2.

2. If the greatest term be 70, the number of terms 21, and the sum of the series 840; what is the least term, and the common difference?—*Ans.* The least term is 10, and the common difference 3.

3. A debt of 135*l.* 4*s.* can be discharged in a year, by weekly payments in arithmetical progression, the least term or payment being 1*s.* what will be the last or greatest payment, and the common difference of the payments?—*Ans.* The common difference will be 2*s.* and the last payment 5*l.* 3*s.*

PROBLEM X.

Given the two extremes and the sum of the series; to find

1. The number of terms.

RULE.

Divide double the sum of the series by the sum of the extremes, and the quotient will be the number of terms.

2. The common difference.

Having now the extremes and the number of terms, the common difference will be found as in problem 7.

$$\text{Thus, } n = \frac{2s}{a + z}, \text{ and } d = \frac{z + a \times z - a}{2s - a - z}.$$

EXAMPLES.

1. If the extremes be 3 and 19, and the sum of the series 99; what is the number of terms, and the common difference? — *Ans.* The number of terms is 9, and the common difference 2.

2. If the extremes be 10 and 70, and the sum of the series 840; what is the number of terms and the common difference? — *Ans.* The number of terms is 21, and the common difference 3.

3. In what time will a debt of 135*l.* 4*s.* be discharged by weekly payments in arithmetical progression, the first payment being 1*s.* and the last 5*l.* 3*s.* and what will be the common difference of the series? — *Ans.* The common difference will be 2*s.* and the debt will be discharged in a year.

PROBLEM XI.

Given the common difference, the number of terms, and the sum of the series; to find the extremes.

RULE.

Divide the sum of the series by the number of terms: then to and from the quotient, add and subtract half the product of the common difference into 1 less than the number of terms, and the sum and difference will be the two extremes.

$$\text{Thus, } a = \frac{s}{n} - \frac{n-1}{2}d, \text{ and } z = \frac{s}{n} + \frac{n-1}{2}d.$$

EXAMPLES.

1. If the common difference be 2, the number of terms 9, and the sum of the series 99; what are the extremes?

Ans. 3 and 19.

2. If the common difference be 3, the number of terms 21, and the sum of the series 840; what are the extremes? — — — *Ans.* 10 and 70.

3. If a debt of 135*l.* 4*s.* can be discharged in a year, by weekly payments in arithmetical progression, whose common difference is 2*s.* what are the first and last payments? — — — *Ans.* 1*s.* and 5*l.* 3*s.*

G E.

GEOMETRICAL PROGRESSION.

A Geometrical progression is a series of numbers, where-
of the succeeding terms are either all greater or less
than their adjacent preceding terms, in such sort that the
ratio or quotient of every two adjacent terms is the same.

NOTE. The same thing is true with respect to the products of the terms of a geometrical proportion as was observed of the sums of the terms of an arithmetical proportion in the note in page 120. And the same analogy holds good in most of their problems: so that many of their rules are almost verbally the same, and differ only in this, that instead of the operations of addition, subtraction, multiplication, and division in arithmetical progression, are required respectively those of multiplication, division, involution, and evolution in geometrical progression.

P R O B L E M I.

From any given term, and with any given ratio, to construct a geometrical series.

R U L E.

For an ascending series multiply the ratio into the first term, the product will be the second; multiply the ratio into the second, the product will be the third; and so on continually multiplying the ratio into the term last found for the next succeeding term. And for a descending series, divide continually by the ratio, for the several succeeding terms.

E X A M P L E.

It is required to raise a geometrical ascending, and also a descending series, from the first term 24, with the ratio 3.

P R O B L E M II.

From any given term of a geometrical series, whose ratio is known, to find any other term, whose distance from the given term is known.

R U L E.

Multiply or divide the given term, by such power of the ratio, whose index is the given distance of the terms, and the product or quotient will be the required term, according as it is to be greater or less than the given term.

Note,

Note, When the power to which the ratio must be raised, is high, the operation will be abbreviated by working according to the 1d note in involution. — Though the chief use of this problem be when the least term is given to find the greatest, it is not confined to them; for it will serve equally well for any other two terms. — See the note to problem 1, in arithmetical progression, for what is meant by the distance of two terms.

EXAMPLES.

1. What is that term of a geometrical progression, whose distance from the term a is expressed by m , the ratio of the series being r ?

$$\text{Ans. } ar^m \text{ or } \frac{a}{r^m}.$$

2. What is the seventh term of the ascending series, whose first term is 1 and ratio 2? — *Ans.* 64.

3. What is the 13th term of the descending series, whose first term is 60, and the ratio $\frac{3}{4}$? — *Ans.* $11\frac{1}{4}$.

4. If 5 be any term of a series whose ratio is 4; what is the term whose distance from it is expressed by 5, the required term being the greater? — *Ans.* 1280.

5. What is the 7th term of the ascending series whose 10th term is 10000 and ratio 3? — *Ans.* $370\frac{1}{27}$.

PROBLEM III.

Between any two given numbers to find any number of geometrical means.

RULE.

Divide the greater number by the less, and extract such root of the quotient as is denominated by 1 more than the number of means required; which root will be the ratio; then from the ratio and one of the given numbers, the means may be found by problem 1.

EXAMPLES.

1. * To find m geometrical means between a and z .

$$\text{First } \sqrt[m+1]{\frac{z}{a}} = \text{the ratio; then the means are } a \times \frac{z}{a}^{\frac{1}{m+1}},$$

* It appears from the solution, that the required means are expressed by the $m+1$ root of the corresponding inter.

$$a \times \sqrt[m+1]{\frac{z}{a}}, a \times \sqrt[m+1]{\frac{z}{a}}, \&c. \text{ unto } a \times \sqrt[m+1]{\frac{z}{a}}; \text{ or, } \sqrt[m+1]{a^m z},$$

$$\sqrt[m+1]{a^{m-1} z^2}, \sqrt[m+1]{a^{m-2} z^3}, \&c. \text{ unto } \sqrt[m+1]{a z^m}.$$

2. To find a mean between 2 and 8. — *Ans.* 4.
3. To find a mean between 1 and 2—*Ans.* 1.414213†.
4. To find two means between 1 and 27. *Ans.* 3 and 9.
5. To find three means between 8 and 128.
Ans. 16, 32, and 64.

PROBLEM IV.

Two terms, with their distance in the series being given; to find any other term, whose distance from either of the given terms is known.

RULE.

intermediate terms of the $\sqrt[m+1]{}$ power of the binomial $a+z$, but wanting their unciæ; and therefore if every term of the series be raised to the $\sqrt[m+1]{}$ power, we shall have a^{m+1} , $a^m z$, $a^{m-1} z^2$, $a^{m-2} z^3$, $a^{m-3} z^4$, &c. unto $(a^{m-m} z^{m+1}) z^{m+1}$, for the terms of the $\sqrt[m+1]{}$ power of $a+z$, without the unciæ. Whence algebraists may solve this problem without finding the ratio; for by involving $a+z$ to the $\sqrt[m+1]{}$ power, the $\sqrt[m+1]{}$ root of each of the intermediate terms, without their unciæ, will be the respective means.—Again, because the similar powers of proportionals are also proportional, the following method of completing an imperfect square naturally presents itself, viz. Divide the second term by the first, multiply the second by the quotient, and $\frac{1}{4}$ of the product will be the third term: thus, if $x^2 + ax = b$; then $\frac{ax}{x^2} = \frac{a}{x}$, and $\frac{1}{4} ax \times \frac{a}{x} = \frac{1}{4} a^2 =$ the third term; and therefore $x^2 + ax + \frac{1}{4} a^2 = b + \frac{1}{4} a^2$; the left-hand side of which equation is a complete square.

R U L E.

Find the ratio, by extracting such root of the quotient of the greater of the given terms divided by the less as is denominated by their distance; then the required term will be found by problem 2.

Note, The same observations are to be understood here as were made in the two notes to problem 4 of arithmetical progression, only using the word quotient instead of difference, and ratio for common difference.

E X A M P L E S.

1. Given the distance n of the two terms a and z , the latter being the greater; to find a term whose distance from a is denoted by m .

First $\sqrt[n]{\frac{z}{a}}$ = the ratio; then $a \times \sqrt[n]{\frac{z}{a}}^m$ = the term required.

Or, If the question be proposed in this manner: Given the m th term a , and the n th term z , of a series; to find the p th term.

Then $\sqrt[n]{\frac{z}{a}}^m$ = the ratio, and $a \times \sqrt[n]{\frac{z}{a}}^{pm}$ = the term required.

2. What is the third term of the series whose first and second terms are 4 and 8? Or, To find a third proportional to the numbers 4 and 8. — — *Ans.* 16.

3. What is the second term of the series whose first and third terms are 4 and 16? Or, To find a mean proportional between the numbers 4 and 16. — *Ans.* 8.

4. What is the tenth term of the series whose fifth term is 81, and eighth term 2187? — *Ans.* 19683.

5. What is the ninth term of the series whose third term is 1, and sixth term $\frac{8}{27}$? — — *Ans.* $\frac{64}{27}$.

6. Given the distance 6 of the two terms 7 and 14, to find a term whose distance from 14 shall be 4; the required term being the greatest. — *Ans.* $14 \times 2^{\frac{2}{3}} = 22.2263 +$.

P R O B L E M V.

Given one of the extremes, the ratio, and the number of the terms of a geometrical series; to find

N

1. The

1. The other extreme.

R U L E.

Multiply or divide the given extreme by such power of the ratio whose index is one less than the number of terms, and the product or quotient will be the required term, according as it is the greater or less extreme.

2. The sum of the series.

R U L E.

Divide the difference of the extremes by the ratio less 1; to the quotient add the greater extreme, and it will give the sum of the series. Or, Multiply the greatest term by the ratio, from the product subtract the least term, then divide the difference by the ratio less 1, and the quotient will be the sum of the series.

Thus, if a represent the least term,

z the greatest,

r the ratio,

n the number of terms, and

s the sum of the series;

$$\text{then } \begin{cases} z = ar^{n-1} \\ a = z \div r^{n-1} \end{cases} \quad \text{and} \quad \begin{cases} s = \frac{r^n - 1}{r - 1} \times a \\ s = \frac{r^n - 1}{r - 1} \times \frac{z}{r^{n-1}} \end{cases}$$

E X A M P L E S.

1. Given the least term 1, the ratio 2, and the number of terms 10; what is the greatest term, and the sum of the series?—*Ans.* The greatest term is 512, and the sum is 1023.

2. If the greatest term be 885735, the ratio 3, and the number of terms 12; what is the least term, and the sum of the series?—*Ans.* The least term is 5, and the sum 1328600.

3. What debt will be discharged in a year or 12 months, by paying 1*l.* the first month, 2*l.* the second, 4*l.* the third, and so on, each succeeding payment being double the last; and what will the last payment be?—*Ans.* The debt is 4095*l.* and the last payment 2048*l.*

PROBLEM VI.

Given the extremes, and the ratio, to find

1. The sum of the series.

This is found by the 2d case of the last problem.

2. The number of terms.

RULE.

Divide the greatest term by the least; find what power of the ratio is equal to the quotient; then add 1 to the index of that power, and the sum will be the number of terms. Or, Divide the difference of the logarithms of the extremes by the logarithm of the ratio; add 1 to the quotient, and the sum will be the number of terms.

$$\text{Thus, } s = \frac{z-a}{r-1} + z = \frac{rz-a}{r-1}, \text{ and } n = \frac{\log z - \log a}{\log r} + 1 = \frac{\log z - \log a + \log r}{\log r}.$$

EXAMPLES.

1. If the extremes be 1 and 512, and the ratio 2; what is the sum of the series, and the number of terms?

Ans. The sum is 1023, and the number of terms 10.

2. If the extremes be 5 and 885735, and the ratio 3; what is the sum of the series, and the number of terms?

Ans. The sum is 1328600, and the number of terms 12.

3. What debt will be discharged by monthly payments in geometrical progression, whereof the first is 1*l.* and the last 2048*l.* the ratio being 2; and in what time will it be discharged?—*Ans.* The debt is 4095*l.* and will be discharged in a year.

PROBLEM VII.

Given the extremes and the number of terms, to find

1. The ratio.

This is found as in problem 4, by dividing the greater extreme by the less, and extracting such root of the quotient whose index is equal to the number of terms minus 1.

2. The sum of the series.

This is found as in problem 5.

$$\text{Thus, } r = \frac{z^{\frac{1}{n-1}}}{a}, \text{ and } s = \frac{z \times \frac{z^{\frac{1}{n-1}}}{a} - a}{\frac{z^{\frac{1}{n-1}}}{a} - 1} = \frac{z^{\frac{n}{n-1}} - a^{\frac{n}{n-1}}}{z^{\frac{1}{n-1}} - a^{\frac{1}{n-1}}}.$$

EXAMPLES.

1. Given the extremes 1 and 512, and the number of terms 10; to find the ratio, and the sum of the series.

Ans. The ratio is 2, and the sum is 1023.

2. If the extremes of a series, consisting of 12 terms, be 5 and 885735; what is the ratio, and the sum of the series?—*Ans.* The ratio is 3, and the sum is 1328600.

3. What debt can be discharged in a year by monthly payments in geometrical progression, whereof the first payment is 1*l.* and the last 2048, and what will the ratio of the series be?—*Ans.* The ratio will be 2, and the debt 4095*l.*

PROBLEM VIII.

Given one of the extremes, the sum of the series, and the ratio; to find

1. The other extreme.

RULE.

To find the greatest term, multiply the difference between the sum of the series and the given term by the ratio minus 1; divide the product by the ratio; to the quotient add the least term, and the sum will be the greatest term.—To find the least term, multiply the difference between the sum of the series and the given term by the ratio minus 1; subtract the product from the greatest term, and the sum will be the least term.

2. The number of terms.

Having now both the extremes and the ratio, the number of terms will be found as in problem 6.

Thus

$$\text{Thus } \begin{cases} z = \frac{s - a \times r - 1}{r} + a = \frac{sr - s + a}{r} \\ a = z - s - z \times r - 1 = zr - sr + s. \end{cases}$$

$$\text{and } \begin{cases} n = \frac{\log. z - \log. zr - sr + s}{\log. r} + 1, \\ n = \frac{\log. sr - s + a - \log. a}{\log. r}. \end{cases}$$

EXAMPLES.

1. Given the least term 1, the sum of the series 1023, and the ratio 2; to find the greatest term and the number of terms.—*Ans.* The greatest term is 512, and the number of terms 10.

2. If the greatest term be 885735, the sum of the series 1328600, and the ratio 3; what is the least term, and the number of terms?—*Ans.* The least term is 5, and the number of terms 12.

3. In what time will a debt of 4095*l.* be discharged by monthly payments in geometrical progression, whereof the first term is 1*l.* and ratio 2; and what will the last payment be?—*Ans.* The last payment will be 2048*l.* and the debt will be discharged in a year.

PROBLEM IX.

Given one extreme, the sum of the series, and the number of terms; to find the other extreme and the ratio.

Thus, the other extreme will be found from this equation,

$$\text{tion, } z \times s - z = a \times s - a, \text{ or } z^{\frac{n}{n-1}} - s z^{\frac{1}{n-1}} = a^{\frac{n}{n-1}} - s a^{\frac{1}{n-1}}; \text{ and the ratio from one of these } r^{\frac{n}{n-1}} - \frac{s}{a} = 1 - \frac{s}{a},$$

$$\text{or } r^{\frac{n}{n-1}} - \frac{s}{s-z} r^{\frac{n-1}{n-1}} = -\frac{z}{s-z}, \text{ according as } a \text{ or } z \text{ is given.}$$

Or, Having found one of those, the other may be found by one of the former problems.

EXAMPLES.

1. Given the least term 1, the sum of the series 1023, and the number of terms 10; to find the greatest term, and the ratio.—*Ans.* The greatest term is 512, and the ratio 2.

2. What is the least term and the ratio of a geometrical series, whose greatest term is 885735, the sum of its terms 1328600, and the number of terms 12?

Ans. The least term is 5, and the ratio 3.

3. If a debt of 4095*l.* can be discharged in a year, by monthly payments in geometrical progression, whose first term is 1*l.* what is the ratio of the series, and what will the last payment be?—*Ans.* The ratio is 2, and the last term 2048*l.*

PROBLEM X.

Given the extremes, and the sum of the series; to find

1. The ratio.

RULE.

Divide the difference between the sum of the series and the least term by the difference between the sum of the series and the greatest term, and the quotient will be the ratio.

2. The number of terms.

This is found by problem 6.

Thus, $r = \frac{s-a}{s-z}$, and $n = \frac{\log. z - \log. a}{\log. s - a - \log. s - z} + 1$.

EXAMPLES.

1. Given the extremes 1 and 512, and the sum of the series 1023; to find the ratio, and the number of terms.

Ans. The ratio is 2, and the number of terms 10.

2. If the extremes be 5 and 885735, and the sum of the series 1328600; what is the ratio, and the number of terms?—*Ans.* The ratio is 3, and the number of terms 12.

3. In what time will a debt of 4095*l.* be discharged by monthly payments in geometrical progression, the first payment being 1*l.* and the last 2048*l.* and what is the ratio of the series?—*Ans.* The ratio is 2, and the debt will be discharged in a year.

PRO-

PROBLEM XI.

Given the ratio, the number of terms, and the sum of the series; to find the extremes.

RULE.

From the ratio subtract 1; raise the ratio to the power denominated by the number of terms, from which subtract 1; divide the first remainder by the last, then multiply the quotient into the sum of the series, and the product will be the least term: from which, together with the ratio, and number of terms, the greatest term will be found by problem 5.

$$\text{Thus } a = \frac{r-1}{r^n-1} \times s, \text{ and } z = \frac{r-1}{r^n-1} \times sr^{n-1}.$$

EXAMPLES.

1. Given the ratio 2, the number of terms 10, and the sum of the series 1023; to find the extremes.

Ans. The extremes are 1 and 512.

2. If the ratio of a series be 3, the number of terms 12, and the sum of the series 1328600; what are the extremes? ——— *Ans.* 5 and 885735.

3. If a debt of 4095*l.* can be discharged in a year, by monthly payments in geometrical progression, the ratio being 2; what will the first and last payment be?

Ans. 1*l.* and 2048*l.*

SINGLE RULE of FALSE POSITION.

THIS rule is called false position (or false supposition) because it makes a supposition of false numbers as if they were the true ones, and by their means discovers the true numbers sought.

The single rule uses only one supposition, but the double rule two; whence come their names.

To the rule of position belong such questions as cannot be solved by a direct process by any of the former rules; and in which the required number or numbers do not ascend above the first power: such, for example, as most of the questions usually brought to exercise the reduction of simple equations in analytics. But it will not
bring

bring out true answers when the numbers sought ascend above the first power; for then the results are not proportional with their positions, nor the errors with the difference of the true number and each position; yet in all such cases it is a very good approximation, and in exponential equations, as well as many other things, succeeds better than perhaps any other method.

Those questions, in which the results are proportional to their suppositions, belong to single position: such are those which require the multiplication or division of the number sought by any number, or where it is to be increased or diminished by itself any number of times, or by any part or parts of it. And those in which the results are not proportional to their positions, belong to the double rule: such are those, in which the number sought is increased or diminished by some given number, which is no known part of the number required.

To work questions in single position.

Take any number, and perform the same operations with it as, in the question, are described to be performed with the number sought; then if the result be the same with that in the question, the supposed number is the number sought; but if it be not, say, As the result of your operation, is to your position, so is the result in the question, to the number required.

Note, 1 may be made a constant supposition in all the questions, and in most cases is better than any other.

EXAMPLES.

1. A person after spending $\frac{1}{3}$ and $\frac{1}{4}$ of his money, has yet remaining 60*l*. what had he at first?

1. Suppose he had at first 120*l*.

Then the $\frac{1}{3}$ of 120 is 40

$\frac{1}{4}$ of it is 30

their sum is 70

which taken from 120

leaves 50

$$\text{And } 50 : 120 :: 60 : \frac{60 \times 120}{50} = \frac{60 \times 12}{5} = 12 \times 12 = 144.$$

2. Suppose he had 11.

Then $\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}$, and $1 - \frac{7}{12} = \frac{5}{12}$; whence
 $\frac{5}{12} : 1 :: 60 : 1 \times 60 \times \frac{12}{5} = \frac{60 \times 12}{5} = 12 \times 12 = 144$, as before.

Proof.

$\frac{1}{3}$ of 144 is 48
 $\frac{1}{4}$ of it is 36
 their sum is 84
 which taken from 144
 leaves 60 as *per* question.

2. What number is that, which multiplied by 7, and the product divided by 6, the quotient may be 14?

Ans. 12.

3. What number is that, which being increased by $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{5}$ of itself, the sum shall be 125? — *Ans.* 60.

4. A general, after sending out a-foraging $\frac{1}{3}$ and $\frac{1}{4}$ of his men, had yet remaining 700; what number had he in command? — *Ans.* 4200.

5. A gentleman distributed 78 pence among a number of poor people, consisting of men, women, and children; to each man he gave 6d. to each woman 4d. and to each child 2d. moreover there were twice so many women as men, and thrice so many children as women; how many were there of each? — *Ans.* 3 men, 6 women, and 18 children.

6. One being asked his age, said, if $\frac{1}{3}$ of the years I have lived, be multiplied by 7, and $\frac{1}{4}$ of them be added to the product, the sum will be 292: what was his age?

Ans. 60 years.

DOUBLE RULE of FALSE POSITION.

HAVING taken any two convenient numbers, for the positions, proceed with each, according to the conditions of the question, as if it was the true number sought, and find how much the results are different from the result in the question; next multiply each of these errors or dif-

differences by the other's position ; then if the errors be of the same affection, that is, if the results be both either too great or too little, divide the difference of the products by the difference of the errors, and the quotient will be the answer ; but if the errors be of different affections, that is, if one result be too great and the other too little, divide the sum of the products by the sum of the errors, and the quotient will be the answer.

Or, Having found the errors, say, As the sum, or difference of the errors, according as they are of a different, or the same kind, is to the difference of the suppositions ; so is the least error, to the correction of the supposition belonging to this error ; which must be added to, or subtracted from it, according to the following conditions ; viz. If the errors be of the same kind, add, or subtract the correction, to, or from this supposition, according as it is greater or less than the other supposition ; but if the errors be of different kinds, add, or subtract, according as this supposition is the less or greater of the two ; and the sum, or difference, will be the number sought.

Note, It is often of advantage to make 1 and 0 the two suppositions.

EXAMPLES.

1. What number is that, which being multiplied by 6, the product increased by 18, and the sum divided by 9, the quotient will be 20 ?

First, suppose 30 to be the number sought ; then $\frac{30 \times 6 + 18}{9} = 10 \times 2 + 2 = 20 + 2 = 22$; but ought to have been 20 ; therefore the error is 2 in excess.

Again, suppose 18 to be the number sought ; then $\frac{18 \times 6 + 18}{9} = 2 \times 6 + 2 = 12 + 2 = 14$; but ought to be 20 ; therefore the error is 6 in defect ; also the errors are of different kinds or affections.

Whence, by the first rule, $\frac{30 \times 6 + 18 \times 2}{2 + 6 = 8} = \frac{15 \times 3 + 9 \times 1}{2}$

$\frac{54}{2} = 27$, the number sought.

And

And by the second rule, $2+6:30-18::2:\frac{2\times12}{8}=3$, the correction; then $30-3=27$, the number sought.

But to work this by the note, suppose first 0; then $\frac{0\times6+18}{9}=\frac{18}{9}=2$; but ought to be 20; therefore the

error is 18 too little. Again, suppose 1; then $\frac{1\times6+18}{9}=2+6$

$\frac{6}{3}=\frac{8}{3}=2\frac{2}{3}$; but should have been 20; therefore the error is $17\frac{1}{3}$ in defect also, and the errors are of the same kind.

Whence, by the first rule, $\frac{0\times17\frac{1}{3}+18\times1}{18-17\frac{1}{3}}=\frac{18}{\frac{1}{3}}=9\times3=27$, the number sought.

And, by the second rule, $18-17\frac{1}{3}:1-0::17\frac{1}{3}:\frac{17\frac{1}{3}}{\frac{1}{3}}=\frac{52}{2}=26$, the correction; then $1+26=27$, the number sought.

2. A son, asking his father how old he was, received the following answer: your age now is $\frac{1}{4}$ of mine; but 5 years ago, your age was only $\frac{1}{5}$ of mine, at that time: what were their ages? — *Ans.* 80 and 20.

3. A workman was hired for 30 days, at 2s. 6d. per day, for every day he worked; but with this condition, that, for every day he played, he should forfeit 1s. Now it so happened, that, upon the whole, he had 2l. 14s. to receive. How many of the days did he work? *Ans.* 24.

4. A and B began to play together with equal sums of money: A first won 20 guineas, but afterwards lost back $\frac{1}{4}$ of what he then had; after which, A had 4 times as much as A. What sum did each begin with?

Ans. 100 guineas.

5. Two persons, A and B, have both the same income: A saves $\frac{1}{5}$ of his; but B, by spending 50l. per annum more than A, at the end of four years finds himself 100l. in debt. What doth each receive and spend per annum? — *Ans.* They receive 125l. per ann. also A spends 100l. and B spends 150l. per annum. A

A PROMISCUOUS COLLECTION of QUESTIONS.

1. **A** was born when B was 21 years of age: How old will A be when B is 47; and what will be the Age of B when A is 60? — *Ans. A 26, B 81.*

2. What difference is there between twice five and twenty, and twice twenty-five? — *Ans. 20.*

3. What number taken from the square of 48 will leave 16 times 54? — — *Ans. 1440.*

4. What number added to the thirty-first part of 3813 will make the sum 200? — — *Ans. 77.*

5. What number deducted from the 23d part of 29440 will leave the 64th part of the same? — *Ans. 820.*

6. The remainder of a division is 325, the quotient 467, the divisor is 43 more than the sum of both; what is the dividend? — — *Ans. 390265.*

7. A person, at the time of his out-setting in trade, owed 350*l.* and had in cash 5307*l.* 10*s.* in wares 713*l.* 7*d.* and in good debts 210*l.* 5*s.* 10*d.* Now after having traded a year he owed 703*l.* 17*s.* and had in cash 4874*l.* 9*s.* 4*d.* in bills 350*l.* in wares 1075*l.* 14*s.* 3½*d.* and in recoverable debts 613*l.* 13*s.* 10½*d.* What was his real gain that year? — — *Ans. 329*l.* 4*s.* 1*d.**

8. Two persons depart from the same place at the same time, the one travels 30, the other 35 miles a day: How far are they distant after 7 days if they travel both the same road, and how far if they travel in contrary directions? — — *Ans. 35 and 455 miles.*

9. A gentleman's daily expence is 4*l.* 8*s.* 1½*d.* and he saves 500*l.* in the year: What is his yearly income? — *Ans. 2107*l.* 12*s.**

10. Having a piece of land 11 poles in breadth, I demand what length of it must be taken to contain an acre, when 4 poles in breadth require 40 poles in length to contain the same? — — *Ans. 14*pls.* 3*yds.**

11. If a gentleman, whose annual income is 1000*l.* spend 21*l.* a week, whether will he save or run in debt, and how much in the year? — *Ans. 92*l.* debt.*

12. In

12. In the latitude of London, the distance round the earth, measuring directly east or west, is about 15550 miles; now as the earth turns round in 23 hours 56 minutes, at what rate *per hour* is the city of London carried by this motion from west to east?

Ans. $649\frac{2}{3}$ miles an hour.

13. In order to raise a joint stock of 10000*l.* A, B, and C, together subscribe 7950*l.* and D the rest: Now A and B are known together to have set their hands to 5800*l.* and A has been heard to say that he had undertaken for 550*l.* more than B. What did each proprietor advance?—*Ans.* A 3175, B 2625, C 2150, D 2050.

14. A tradesman increased his estate annually by 100*l.* more than $\frac{1}{4}$ part of it, and at the end of 4 years found that his estate amounted to 10342*l.* 3*s.* 9*d.* What had he at out-setting? — — *Ans.* 4000*l.*

15. Paid 1012*l.* 10*s.* for 750*l.* taken in 7 years ago; at what rate *per cent. per ann.* did I pay interest? *Ans.* 5*l.*

16. What is the interest of 720*l.* for 73 days, at 3*l.* *per cent. per annum*? — — *Ans.* 4*l.* 6*s.* 4*d.* $3\frac{1}{8}\frac{6}{11}\frac{7}{11}$.

17. Part 1200 acres of land among A, B, and C, so that B may have 100 more than A, and C 64 more than B. — — *Ans.* A 312, B 412, C 476.

18. Divide 1000 crowns, give A 120 more and B 96 less than C. — — *Ans.* A 445, B 230, C 325.

19. To how much amounts the order, for which my factor, at the rate of $2\frac{1}{2}$ *per cent.* receives 22*l.* 10*s.*

Ans. 900*l.*

20. What sum of money will amount to 132*l.* 16*s.* 3*d.* in 15 months, at 5 *per cent. per annum* simple interest?

Ans. 125*l.*

21. Laid out 165*l.* 15*s.* in wine at 4*s.* 3*d.* a gallon; some of which receiving damage in carriage, I sold the rest at 6*s.* 4*d.* a gallon, which produced only 110*l.* 16*s.* 8*d.* What quantity was damaged? — *Ans.* 430 gal.

22. A father divided his fortune among his sons, giving A 4 as often as B 3, and C 5 as often as B 6; what was the whole legacy, supposing A's share were 5000*l.*

Ans. 11875*l.*

23. A stationer sold quilts at 10s. 6d. a thousand, by which he cleared $\frac{1}{7}$ of the money; but growing scarce, raised them to 12s. a thousand; what did he clear *per cent.* by the latter price? — *Ans.* 71l. 8s. 6 $\frac{1}{2}$ d.

24. If 1000 men, besieged in a town, with provisions for 5 weeks, allowing each man 16 oz. a day, were reinforced with 500 men more; and hearing that they cannot be relieved till the end of 8 weeks; how many ounces a day must each man have, that the provision may last that time? — — — *Ans.* 6 $\frac{2}{3}$ oz.

25. If a quantity of provisions serve 1500 men 12 weeks, at the rate of 20 ounces a day for each man; how many men will the same provisions maintain for 20 weeks, at the rate of 8 oz. a day for each man? — *Ans.* 2250 men.

26. In what time will the interest of 72l. 12s. equal that of 15l. 5s. for 64 days, at any rate of interest? — *Ans.* 13 $\frac{1}{3}$ $\frac{6}{11}$ days.

27. A person possessed of $\frac{1}{3}$ of a ship, sold $\frac{2}{3}$ of his share for 1260l. what was the reputed value of the whole at the same rate? — — — *Ans.* 5040l.

28. What sum of money at 4 $\frac{1}{2}$ *per cent.* will clear 29l. 15s. in a year and a half's time? — *Ans.* 440l. 14s. 9 $\frac{1}{2}$ d.

29. What number is that, to which if $\frac{2}{7}$ of $\frac{5}{6}$ be added, the sum will be 1? — — — *Ans.* $\frac{5}{6}\frac{1}{3}$.

30. A father dying, left his son a fortune, $\frac{1}{4}$ of which he ran through in 8 months; $\frac{3}{7}$ of the remainder lasted him a twelve-month longer, after which he had bare 410l. left: What did his father bequeath him? — *Ans.* 950l. 13s. 4d.

31. Bought a quantity of goods for 250l. and 3 months after sold it for 275l. How much *per cent. per annum* did I gain by them? — — — *Ans.* 40l.

32. A guardian paid his ward 3500l. for 2500l. which he had had in his hands 8 years: What rate of interest did he allow him? — — — *Ans.* 5 *per cent.*

33. Bought a quantity of goods for 150l. ready money, and sold it again for 200l. payable at the end of 9 months; what was the gain in ready money, supposing rebate to be made at 5 *per cent.* — *Ans.* 42l. 15s. 5 $\frac{1}{4}$ d.

34. A

34. A person being asked the hour of the day, said, The time past noon is equal to $\frac{4}{7}$ ths of the time till midnight: What was the time? — *Ans.* 20 min. past 5.

35. A person, looking on his watch, was asked what was the time of the day, who answered, It is between 4 and 5; but a more particular answer being required, he said that the hour and minute hands were then exactly together: What was the time? — *Ans.* 21 $\frac{9}{11}$ min. past 4.

36. With 12 gallons of canary at 6s. 4d. a gal. I mixed 18 gal. of white-wine at 4s. 10d. a gal. and 12 gal. of cyder at 3s. 1d. a gal. At what rate must I sell a quart of this composition so as to clear 10 per cent? *Ans.* 5s. 2 $\frac{1}{2}$ d.

37. Suppose that I have $\frac{3}{8}$ of a ship worth 1200l. what part of her have I left after selling $\frac{2}{3}$ of $\frac{4}{5}$ of my share, and what is it worth? — *Ans.* $\frac{1}{4}$ $\frac{1}{8}$ worth 185l.

38. What length must be cut off a board 8 $\frac{3}{8}$ inches broad, to contain a square foot, or as much as 12 inches in length and 12 in breadth? — *Ans.* 17 $\frac{1}{8}$ inches.

39. What sum of money will produce as much interest in 3 $\frac{1}{2}$ years, as 210l. 3s. can produce in 5 years and 5 months? — — — *Ans.* 350l. 5s.

40. There is gained by trading with a ship 120l. 14s. Now suppose that $\frac{1}{4}$ of her belongs to S, $\frac{1}{8}$ to T, $\frac{1}{8}$ to V, and the rest to W; what must each have of the gain? — *Ans.* S 30l. 3s. 6d. T 45l. 5s. 3d. V 15l. 1s. 9d. W 30l. 3s. 6d.

41. If 100l. in 5 years be allowed to gain 20l. 10s. in what time will any sum of money double itself at the same rate of interest? — — — *Ans.* 24 $\frac{1}{4}$ years.

42. What difference is there between the interest of 350l. at 4 per cent. for 8 years, and the discount of the same sum, at the same rate, and for the same time? — — — *Ans.* 27l. 3 $\frac{1}{4}$ s.

43. If, by selling goods at 50s. per cwt. I gain 20 per cent. what do I gain or lose per cent. by selling at 45s. per cwt? — — — — — *Ans.* 8l. gain.

44. If, by remitting to Holland, at 34s. 6d per l. sterling, 4 $\frac{1}{2}$ per cent. be gained; how goes the exchange, when by remittance I clear 10 per cent? *Ans.* 36s. 3 $\frac{1}{8}$ d.

45. Sold goods for 60 guineas, and by so doing, lost 17 *per cent.* whereas I ought, in dealing, to have cleared 20 *per cent.* Then how much under their just value were they sold? — — *Ans.* 28*l.* 1*s.* 8 $\frac{3}{8}$ *d.*

46. If, by selling goods at 27*d.* *per lb.* I gain *cent. per cent.* what do I clear *per cent.* by selling for 9 guineas *per cwt*? — — — *Ans.* 50 *per cent.*

47. If 20 men can perform a piece of work in 12 days, how many will accomplish another thrice as big in one-fifth of the time? — — — — *Ans.* 300.

48. A younger brother received 2430*l.* which was just $\frac{3}{4}$ of his elder brother's fortune; and 2 and $\frac{2}{7}$ times the elder's money was half as much again as the father was worth: What was that? — — *Ans.* 14400*l.*

49. A person making his will, gave to one child $\frac{1}{5}$ of his estate, and the rest to another; and when these legacies came to be paid, the one turned out 600*l.* more than the other: What did the testator die worth?

Ans. 2000*l.*

50. A father devised $\frac{7}{8}$ of his estate to one of his sons, and $\frac{7}{8}$ of the residue to another, and the surplus to his relict for life: the children's legacies were found to be 257*l.* 3*s.* 4*d.* different: Pray what money did he leave the widow the use of? — — *Ans.* 635*l.* 10 $\frac{3}{4}$ *s.*

51. What number is that, from which, if you take $\frac{2}{3}$ of $\frac{1}{8}$, and to the remainder add $\frac{7}{8}$ of $\frac{1}{10}$, the sum will be 10? — — — *Ans.* 10 $\frac{191}{240}$.

52. There is a number which, if multiplied by $\frac{2}{3}$ of $\frac{7}{8}$ of $1\frac{1}{2}$, will produce 1: What is the square of that number?

Ans. $1\frac{1}{4}$.

53. A person dying, left his wife with child, and making his will, ordered that if she went with a son, $\frac{2}{3}$ of his estate should belong to him, and the remainder to his mother; and if she went with a daughter, he appointed the mother $\frac{2}{3}$ and the girl the remainder: but it happened that she was delivered both of a son and daughter; by which she lost in equity 2400*l.* more than if it had been only a girl: what would have been her dowry had she had only a son? — — — *Ans.* 2100*l.*

54. Three

54. Three persons purchase together a ship, towards the payment of which A advanced $\frac{2}{3}$, and B $\frac{1}{3}$ of the value, and C 200*l.* How much paid A and B, and what part of the vessel had C?

Ans. A 90 $\frac{1}{3}$ *l.* B 116 $\frac{4}{3}$ *l.* C $\frac{3}{8}$ part.

55. A and B clear by an adventure at sea, 60 guineas, with which they agree to buy a horse and chaise, of which they were to have the use, in proportion to the sums adventured, which was found to be A 9 to B 8; they cleared 45 per cent. What money then did each send abroad?

Ans. A 74*l.* 2*s.* 4 $\frac{1}{4}$ *d.* and B 65*l.* 17*s.* 7 $\frac{1}{4}$ *d.*

56. In an article of trade, A gains 18*s.* 3*d.* and his adventure was 40*s.* more than B's, whose share of profit is but 12*s.* What are the particulars of their stock?

Ans. A 5*l.* 16*s.* 9 $\frac{3}{4}$ *d.* and B 3*l.* 16*s.* 9 $\frac{3}{4}$ *d.*

57. Three persons entered joint trade, to which A contributed 240*l.* and B 210*l.* they clear 120*l.* of which 30*l.* belongs of right to C. Required that person's stock, and the several gains of the other two?

Ans. C's stock 150*l.* A gained 48*l.* and B 42*l.*

58. A and B in partnership equally divide the gain; A's money, which was 96*l.* 12*s.* lay for 15 months, and B's for no more than 6: What was the adventure of the latter? — — — *Ans.* 24*l.* 10*s.*

59. Put out 420*l.* to interest, and in 6 $\frac{1}{2}$ years time there was found to be due 556*l.* 10*s.* What was the rate of interest? — — — *Ans.* 5 per cent.

60. A clears 12*l.* in 6 months, B 15*l.* in 5 months, and C, whose stock was 40*l.* clears 21*l.* in 9 months: What was the whole stock? — — — *Ans.* 125 $\frac{5}{7}$ *l.*

61. A had 12 pipes of wine, which he parted with to B at 4 $\frac{1}{2}$ per cent. profit, who sold them to C for 40*l.* 12*s.* advantage; C made them over to D for 605*l.* 10*s.* and cleared thereby 6 per cent. How much a gallon did this wine cost A? — — — *Ans.* 6*s.* 8 $\frac{26}{100}$ *d.*

62. A, of Amsterdam, orders B of London, to remit to C of Paris, at 52 $\frac{1}{2}$ *d.* ster. a crown, and to draw on P, of Antwerp, for the value, at 34 $\frac{1}{2}$ *s.* Flem. a *l.* ster. but as soon as B received the commission, the exchange was on

Paris at 53*d.* a crown: Pray at what rate of exchange ought B to draw on C, to execute his orders, and be no loser? ——— *Ans.* 34*s.* 2 $\frac{5}{11}$ *d.*

63. A, with intention to clear 20 guineas, on a bargain with B, rates hops at 15*d.* a *lb.* which cost him 10 $\frac{1}{2}$ *d.* B, apprized of that, sets down malt, which cost 20*s.* a quarter, at an adequate price: For how much malt did they contract? ——— *Ans.* 49 *qrs.*

64. A and B venturing equal sums of money, clear by joint trade 180*l.* By agreement, A was to have 8 *per cent.* because he spent time in the execution of the project, and B was to have only 5: What was allotted to A for his trouble? ——— *Ans.* 41*l.* 10*s.* 9 $\frac{1}{3}$ *d.*

65. Laid out in a lot of muslin 500*l.* upon examination of which, 3 parts in 9 proved damaged, so that I could make but 5*s.* a yard of the same; and by so doing find I lost 50*l.* by it. At what rate per ell am I to part with the undamaged muslin, in order to gain 50*l.* upon the whole? ——— *Ans.* 11*s.* 7 $\frac{1}{2}$ *d.*

66. A, at Paris, draws on B, in London, 1400 crowns, at 56*d.* sterl. a crown, for the value of which B draws again on A at 57*d.* sterl. a crown, besides reckoning commission $\frac{1}{2}$ *per cent.* Did A gain or lose by this transaction, and what? ——— *Ans.* He gained 17 $\frac{3}{4}$ crowns.

67. A, B, and C, are in company; A put in his share of the stock for 6 months, and laid claim to $\frac{1}{3}$ of the profits; B put in his for 9 months; C advanced 500*l.* for 8 months, and required on the balance $\frac{1}{3}$ of the gain: Required the stock of the other two adventurers.

Ans. A 185*l.* 3*s.* 8 $\frac{4}{9}$ *d.* and B 172*l.* 16*s.* 9 $\frac{1}{4}$ *d.*

68. A young hare starts 40 yards before a greyhound, and is not perceived by him till she has been up 40 seconds; she scuds away at the rate of 10 miles an hour, and the dog, on view, makes after her at the rate of 18; how long will the course hold, and what ground will be run over, beginning with the out setting of the dog?

Ans. 60 $\frac{5}{11}$ *sec.* and 530 yards run.

69. If I leave Exeter at 8 o'clock on monday morning for London, and ride at the rate of 3 miles an hour with

out

out intermission; and you set out from London for Exeter at 4 the same evening, and ride 4 miles an hour constantly: Supposing the distance between the two cities to be 130 miles, whereabouts on the road shall you and I meet? —

Ans. $69\frac{1}{2}$ miles from Exeter.

70. A reservoir for water has two cocks to supply it; by the first alone it may be filled in 40 minutes, by the second in 50 min. and it hath a discharging cock, by which it may, when full, be emptied in 25 min. Now, supposing that these 3 cocks are all left open, and that the water comes in; in what time, supposing the influx and efflux of the water to be always alike, would this cistern be filled? — — —

Ans. 3 hrs. 20 min.

71. A sets out of London for Lincoln, at the very same time that B at Lincoln sets forward for London, distant 100 miles: After 7 hours they meet on the road, and it then appeared that A had rode $1\frac{1}{2}$ miles an hour more than B. At what rate an hour did each of them travel?

Ans. A $7\frac{5}{8}$, and B $6\frac{1}{8}$ miles.

72. A and B truck, A has $12\frac{1}{2}$ cwt. of Farnham hops, at 2*l.* 16*s.* a cwt. but in barter insists on 3*l.* B has wine worth 5*s.* a gal. which he raises in proportion to A's demand. On the ballance A received but a *hhd.* of wine: What had he in ready money? —

Ans. 20*l.* 12*s.* 6*d.*

73. A, of Amsterdam, owes to B, of Paris, 3000 guilders of current specie, which he is to remit to him, by order, the exchange 9*d.* flem. *de banco* a crown, the agio 4 *per cent.* but when this was to be negotiated, the exchange was down at 90*d.* a crown, and the agio 5 *per cent.* What did B get by this turn of affairs.

Ans. 5 *liv.* 12 *fol.* $8\frac{584}{1153}$ *den.*

An APPENDIX,

Containing a Course of Book-keeping according to the method of Single Entry, with a Description of the Books, and Directions for using them: Very useful either for young Book-keepers entering into Business, or for Masters to teach in their Schools.

IT is very necessary that almost every person who is intended for business, should learn a course of book-keeping of this kind, because it is used in almost every shop. The italian method alone is not sufficient; for it is a constant complaint among the merchants, &c. who use this method, that their boys, having learnt only the italian method, when they first come to business, are almost as ignorant in the management of their books as if they had never learnt any method. There are some boys who have not time to learn, or, perhaps, a capacity to understand a compleat course of the italian method; there are also many intended for such kinds of business, as that the italian method would be thrown away upon them; to all such, then, this method will be extremely useful. And even supposing a boy were intended for a business which requires the italian method alone, I would, notwithstanding, have him taught this method first, if it were only to facilitate his acquisition of the other. This method is so easy, that it may also be taught in a few weeks' time to young ladies as well as young gentlemen.

The forms of the books may be sufficiently known by inspection — In the day-book, every person is wrote down *Dr. to* the things he receives from you on trust, and *Cr. by* those which you receive from him. In the margin of the day-book are wrote the pages where the accounts stand in the ledger: Instead of these marginal figures, some make only a stroke with the pen, to shew that the account has been posted, that is, entered in the ledger; but it is better to use the figures, for they shew, not only that the account has been posted, but likewise where to find it in the ledger without looking in the alphabet. In the day-book

book I have cast up all the accounts, as I think it better to do so than to leave them uncalculated till the posting; for, as a bill-of-parcels is generally sent along with the goods, if you do not enter the amounts in the day-book, you will frequently have the mortification to find that your book-debts and bills-of-parcels will not agree; for calculations of the same accounts made at different times will sometimes differ.

I have entered in the day-book what is received as well as what is delivered, which is absolutely necessary in teaching; for the learner ought to make out all his own ledger from his day-book.

There are several other books kept by most merchants, as the cash book, the book of house expences, the invoice book, &c.

Directions for the learner.

Having ruled your books in the proper form, copy into your day-book one month's accounts, then calculate them upon your slate or waste-paper, to find if they be rightly cast up, and to exercise you in calculations. Next rule your slate in the form of the ledger, and posted upon it the accounts entered in the day-book, with their dates prefixed, observing to put on the Dr. side of each person's account, those accounts to which he is Dr. in the day-book, and on the Cr. side those by which he is Cr. and if any account consist but of one article, you are to express it particularly with its money in the columns; but if of several, write *to* or *by* sundries, placing the sum of the amounts of all the articles in the columns. After the accounts are, by correcting if necessary, placed according to the teacher's mind, transcribe them into your ledger, leaving a proper space under each person's name to receive more accounts. Then under the proper letters in the alphabet enter those names with the pages where they stand in the ledger; and lastly, write the ledger pages to the several accounts in the day-book. Do the same with the next month's accounts; &c. till the whole be finished. —But observe that you must not enter any person's name
down

down again which has been entered before till the space, first assigned to it, shall be filled with articles; and then the account must be transferred to a new place, as Lady Strawberry's account is from fol. 1 to fol. 5.

When the first ledger, titled A, is filled with accounts, you must, as is done with the following ledger, transfer the unbalanced accounts to the second ledger, titled B, &c. according to the order of the letters of the alphabet; and at the end of the old ledger draw out a balance account, placing your debts on one side, and your credits on the other.

D A Y - B O O K.

January 1, 1766.		£.	s.	d.
1	<i>Mr. James Elford, of Bath, Dr.</i>			
	<i>s. d.</i>			
	To 15 yds. of fine broad cloth, at 13 6	10	2	6
	— 24 — superfine — 18 9	22	10	-
		<hr/>	<hr/>	<hr/>
		32	12	6
<hr/>				
1	<i>Dr Tristram Shandy, of York, Dr.</i>			
	<i>s. d.</i>			
	To 12 gal. palm-sack, at — 8 6	5	2	-
	— 17 — port, red — 5 8	4	16	4
	— 9 — claret — 8 9	3	18	9
		<hr/>	<hr/>	<hr/>
		13	17	1
<hr/>				
	4			
1	<i>Mrs. Mary Masterman, Dr.</i>			
	<i>s. d.</i>			
	To 1 $\frac{1}{2}$ lb. green tea, — at 16 —	1	4	-
	— 2 $\frac{1}{4}$ congou, — 9 6	1	1	4 $\frac{1}{2}$
	— $\frac{1}{4}$ stone of sugar — 5 —	-	1	3
	— A lump of sugar, wt. 20 $\frac{1}{2}$ lb. at — 8	-	13	8
		<hr/>	<hr/>	<hr/>
		3	-	3 $\frac{1}{2}$
				Ja-

January 9					£.	s.	d.
1	<i>Lady Strawberry, Dr.</i>				s.	d.	
	To 9½ yds. of silk,	—	at	12 9	6	1	1½
	— 13 — flowered ditto,	—	—	15 6	10	1	6
					16	2	7½
20							
1	<i>Sir Jonas Moore, Dr.</i>						
	To a ream of thick post paper				1	-	-
27							
2	<i>Mr. James Wilson, Schoolmaster, Dr.</i>				s.	d.	
	To 6 schoolmaster's guides,	at	1 4½		-	8	3
	— 3 doz. copy books,	—	2 6		-	7	6
	— 2 quires foolscap,	—	- 10		-	1	8
	— 1 quire thin post,	—	—		-	1	-
					-	18	5
Feb. 5							
2	<i>Mr. Alderman Ableman, Dr.</i>				l.	s.	d.
	To a ledger ruled,	—			-	15	-
	— 5 C quilts,	—	at - 2 6		-	12	6
	— 3 reams thick post,	—	1 - -		3	-	-
	— 6 quires pot	—	- - 8		-	4	-
	— 40 reams blue demy	—	- 5 6		11	-	-
	— 2 penknives and an inkstand				-	6	-
					15	17	6
12							
2	<i>William Winton, Esq; Dr.</i>				s.	d.	
	To 20 oz. of nutmegs,	—	at - 3		-	5	-
	— 5½ lb. coffee,	—	— 4 -		1	2	-
	— 3¼ — cocoa,	—	— 2 4		-	7	7
	— 4 — almonds	—	— 1 -		-	4	-
	— 8½ — raisins	—	— - 7		-	4	11½
					2	3	6½
							Feb.

Feb. 20

1	<i>Sir Jonas Moore, Cr.</i>			£	s.	d.
	By Cash received of him in full			1	-	-
	27					
2	<i>Sir Jeffery Slingstone, Dr.</i>					
		oz. dwt. gr.	s. d.			
	To a silver punch-bowl	wt. 23 4 —	at 5 10	6	15	4
	— a tankard	10 3 6	-- 6 2	3	2	8
	— a tea-pot and lamp	30 5 12	-- 7 3	10	19	5 $\frac{1}{2}$
	— 6 plates	73 11 5	-- 6 1	22	7	5 $\frac{1}{2}$
	— 18 spoons	41 — 10	-- 6 3	12	16	4 $\frac{1}{2}$
				56	1	4

March 10

2	<i>Mr. William Watson, Dr.</i>					
			s. d.			
	To 2 gal. rum	—	at 10 —	1	-	-
	— 4 — brandy	—	— 10 6	2	2	-
	— 3 — eng. gin	—	— 5 —	-	15	-
				3	17	-

22

1	<i>Doctor Tristram Shandy, of York, Dr.</i>					
			s. d.			
	To 27 $\frac{3}{4}$ gal. sherry	at	6 2	8	11	1 $\frac{1}{2}$
	— 22 $\frac{1}{2}$ — rhenish	—	6 4	7	2	6
	— 34 — Lisbon	—	4 10	8	4	4
				23	17	11 $\frac{1}{2}$

April 7

3	<i>Sir Thomas Lawson, Dr.</i>					
			s. d.			
	To 7 $\frac{1}{2}$ yds. of scarlet cloth	at	21 —	7	17	6
	— 4 superfine blue	—	20 —	4	-	-
	— $\frac{1}{4}$ velvet	—	18 —	-	4	6
	— 30 gold lace	—	10 6	15	15	-
				27	17	-

April

APPENDIX.

157

April 12				£.	s.	d.
<i>Lady Strawberry, Dr.</i>						
			s. d.			
To	11 $\frac{1}{4}$ yds. lustring	at	6 10	4	-	3 $\frac{1}{2}$
—	14 — brocade	—	11 3	7	17	6
				11	17	9 $\frac{1}{2}$
24						
<i>David Johnson, Esq; Dr.</i>						
			s. d.			
To	5 gal. lamp oil	at	4 2	1	-	10
—	3 $\frac{1}{2}$ — train oil	—	3 -	-	10	6
—	$\frac{1}{4}$ — sweet oil	—	12 6	-	9	4 $\frac{1}{2}$
				2	-	8 $\frac{1}{2}$
25						
<i>Mrs. Mary Masterman, Cr.</i>						
By cash, received of her in full				3	-	3 $\frac{1}{2}$
May 3						
<i>Mr James Elford, of Bath, Dr.</i>						
			s. d.			
To	27 yds. of yard-wide cloth	at	8 4	11	5	-
—	16 — drugget	—	6 3	5	-	-
—	12 — serge	—	2 10	1	14	-
—	32 — shalloon	—	1 8	2	13	4
				20	12	4
10						
<i>Sir Thomas Lawson, Dr.</i>						
			s. d.			
To	7 yds. superfine black cloth	at	19 6	6	16	6
—	12 — shalloon	—	2 4	1	8	-
—	1 doz. and 9 coat buttons		2 6	-	4	4 $\frac{1}{2}$
—	2 — 8 waistcoat ditto		1 3	-	3	4
				8	12	2 $\frac{1}{2}$

May 14				£.	s.	d.	
3	Mr. Nicholas Norton, of Durham, Dr.						
			s. d.				
	To	9 pair worsted stockings at	4 6	2	-	6	
	—	6 — silk ditto	15 9	4	14	6	
	—	17 — thread	5 4	4	10	8	
	—	23 — cotton	4 10	5	11	2	
	—	14 — yarn	2 4	1	12	8	
	—	18 — women's gloves	4 2	3	15	-	
	—	19 yds of flannel	1 7½	1	10	10½	
				23	15	4½	
20							
3	David Johnson, Esq; Dr.						
		c. qr. lb.	£. s. d.				
	To	13 cheshire } wt. 5 3 12 at 1 12 6		9	10	4¼	
		cheeses }					
	—	25 glocester — 3 - 18 — 1 8 -		4	8	6	
	—	47 skilton — 1 2 5 — 2 4 8		3	8	11¼	
				17	7	10	
26							
3	Mrs. Shields, Dr.						
	To	8 lb. rice	at 4½	-	3	-	
	—	3½ — currants	5	-	1	5½	
	—	2 quarts of vinegar	6	-	1	-	
				-	5	5½	
June 3							
3	Sir Thomas Lawson, Cr.						
	By a bill on captain James Dixon				10	-	-
3	Capt. James Dixon, Dr.						
		qr. bush.	£. s. d.				
	To	7 3 of wheat	at 1 8 -	10	(6	
	—	9 7 — rye	1 1 6	10	1	3¼	
	—	17 4 — oats	- 10 8	9	(8	
				30	5	5½	
						June	

APPENDIX.

159

June 12		£	s.	d.
2	<i>Sir Jeffery Slingstone, Cr.</i>			
	By a bank note received by the servant	20	-	-
17				
1	<i>Mrs. Mary Masterman, Dr.</i>			
		s.	d.	
	To 14 lb. hard soap at	-	6	-
	— 7 — soft —	-	5	-
	— 3½ — starch —	-	5½	-
	— 3½ — blue —	1	4	-
	— 40 — raisins —	-	4½	-
	— 3 doz. candles —	5	9	-
		2	8	5½
21				
1	<i>Mrs. Masterman, Cr.</i>			
		s.	d.	
	By 40 yds. russia sheeting at	2	2	
28				
3	<i>David Johnson, Esq; Dr.</i>			
		£.	s.	d.
	To 17 lb. cream cheese at	-	-	7½
	— 53 ston 3 lb. bacon —	-	4	8
	— 15½ firks. butter —	1	8	-
		34	12	11½
July 3				
4	<i>Miss Fanny Dawson, of Liverpool, Dr.</i>			
		s.	d.	
	To 14 yds. blue ribbon at	-	7½	-
	— 21 — white —	-	6	-
	— 12½ — lace —	3	6	-
	— 9 pair kid gloves —	2	4	-
		4	4	-
7				
2	<i>Mr. James Wilson, Schoolmaster, Cr.</i>			
	By cash, received in full	-	18	5
				July

July 10					
<i>Mr. Roger Retail, of Newcastle upon Tyne,</i>			£.	s.	d.
4	<i>Dr.</i>				
		s. d.			
	To 24½ lb. royal green tea at	18 6	22	13	3
	— 21¼ — imperial ———	24 -	25	10	-
	— 35¾ — best bohea ———	13 10	24	14	6½
	— 17¾ — coffee ———	5 4	4	14	2½
	— 25 — double refin'd sugar	1 1½	1	8	1½
	— 9 sugar loaves, wt. 137 lb. at	- 7½	4	5	7½
			83	5	9
17					
4	<i>Mr Charles Anderson, Dr.</i>				
		s. d.			
	To 6 mahogany chairs at	18 6	5	11	-
	— 2 elbow ditto ———	25 -	2	10	-
	— 2 pier glasses ———	36 -	3	12	-
			11	13	-
24					
4	<i>Mr. Charles Anderson, Dr.</i>				
		s. d.			
	To 25 yds. curtain stuff at	2 2	2	14	2
	— 12 — ticking ———	1 3	-	15	-
	— 3 stone of feathers ———	25 -	3	15	-
	— 2 pier tables ———	50 -	5	-	-
			12	4	2
28					
3	<i>Capt. James Dixon, Dr.</i>				
		s. d.			
	To 12 bush. peas — at	2 9	1	13	-
	— 9 — beans ———	3 5	1	10	9
	— 17 — malt ———	4 8	3	19	4
	— 25 lb. hops ———	1 4	1	13	4
			8	16	5

August 1				£.	s.	d.
2	<i>William Winton, Esq; Dr.</i>					
			s. d.			
	To 10 gross of bottles	at	22 -	11	-	-
	— 9 — small ditto	—	15 -	6	15	-
	— 2 doz. wine glasses	—	4 6	-	9	-
	— 3 decanters	—	1 2	-	3	6
				18	7	6
7						
2	<i>Mr. Alderman Ableman, Cr.</i>					
	By a note upon Dr. James for			10	-	-
	— cash, in full			5	17	6
				15	17	6
12						
4	<i>Mr. Charles Anderson Dr.</i>					
			s. d.			
	To a Mahogany bed-stead			2	10	-
	2 stools — at	5 3		-	10	6
	poker, tongs, and fender			1	-	-
	2 other sets of irons at	15 -		1	10	-
				5	10	6
16						
3	<i>David Johnson, Esq; Cr.</i>					
	By cash, in part			50	-	-
18						
5	<i>Mr. Conrade Compound, of Exeter, Dr.</i>					
			s. d.			
	To 2 1 $\frac{1}{8}$ lb. cochineal	at	29 6	31	10	5 $\frac{3}{4}$
	6 $\frac{1}{4}$ — opium	—	6 4	1	19	7
	53 $\frac{1}{8}$ — scammony	—	8 10	23	9	3 $\frac{1}{4}$
				56	19	5
21						
4	<i>Mr. Charles Anderson, Cr.</i>					
	By 5 pockets of hops, at 48 s.			12	-	-

August 26			£.	s.	d.
4	<i>Mr. John Baker, Dr.</i>				
		<i>s. d.</i>			
	To 5 gross of brass buttons at 18	-	4	10	-
	— 2 — white — 15	-	1	10	-
	— 7 doz. pair of buckles — 2 2 a pair		9	2	-
	— 12 trunk locks — - 10 each		-	10	-
	— 6 chamber ditto — 2 6		-	15	-
			16	7	-
Sept. 3					
5	<i>Mrs. March, Dr.</i>				
	To 8 farcenet hoods, at 4 3		1	14	-
4					
1	<i>Lady Strawberry, Dr.</i>				
	To 12 $\frac{1}{4}$ yds. fatten at 10 8		6	10	8
6					
2	<i>Mr. James Wilson, Schoolmaster, Dr.</i>				
		<i>s. d.</i>			
	To 6 schoolmaster's guides at 1 4 $\frac{1}{2}$		-	8	3
	— 1 thous. pinions		-	2	6
	— 3 doz. copy-books — 2 6		-	7	6
	— 3 quires of thin post — 1 -		-	3	-
	— Dr Lowth's eng. grammar		-	2	4
			1	3	7
9					
3	<i>Mr. Nicholas Norton, Cr.</i>				
	By a bank note for — —		20	-	-
12					
1	<i>Lady Strawberry, Dr.</i>				
	To 11 $\frac{1}{8}$ yds. of velvet at 18 -		10	4	9
16					
2	<i>Mr. James Wilson, Schoolmaster, Dr.</i>				
	To the universal penman —		1	5	-
16					
5	<i>Mrs March, of Chester, Dr.</i>				
	To 17 india fans at 3s. 10d.		3	5	12
					Sept.

APPENDIX.

163

Sept. 18		£.	s.	d.
1	<i>Mrs. Mary Masterman, Dr.</i>			
	To cash, in full	1	18	2 $\frac{3}{4}$
	22			
1	<i>Lady Strawberry, Cr.</i>			
	By cash, received by the Steward	20	-	-
	24			
4	<i>Mr. Charles Anderson, Cr.</i>			
	By cash, in full	17	7	8
	29			
5	<i>Mrs. March, Dr.</i>			
	To 21 yds. silver ribbon at 2 2	2	5	6
	11 $\frac{1}{2}$ — fine lace — 10 6	6	-	9
		8	6	3
	Oct. 2			
5	<i>Mr. Samuel Edwards, Dr.</i>			
	To 14 lb. of flax at 1s.	-	14	-
	4			
5	<i>Mr. R. Barber, Bristol, Stationer, Cr.</i>			
	By 30 reams of foolscap paper at 12s. 6d.	18	15	-
	6			
5	<i>Lady Strawberry, Dr.</i>			
	To 27 $\frac{1}{2}$ yds. of holland at 5s. 6d.	7	11	3
3	<i>David Johnson, Esq; Cr.</i>			
	By cash, in full	4	1	6
	10			
5	<i>Mr. Matthew Milton, of Norwich, Dr.</i>			
	To 40 ells of dowlas at 1 6	3	-	-
	34 ——— diaper — 1 4 $\frac{1}{2}$	2	6	9
	31 ——— holland — 5 8	8	15	8
		14	2	5
	13			
5	<i>Lady Strawberry, Dr.</i>			
	To 40 yds. of irish cloth at 3s. 4d.	6	13	4
				Oct.

		Oct. 15			£.	s.	d.
6	<i>Mr. Henry Forster, Dr.</i>						
	To 2½ cwt. of iron at 18s. 9d.				2	6	10½
		21					
6	<i>Mrs. Mary Gray, Cr.</i>						
	By 3 ps. irish cloth, quant. 87 yds. at 2s. 2d.				9	8	6
		23					
4	<i>Mr. John Baker, Cr.</i>						
	By cash, in part				10	-	-
		Mrs. March, Dr.					
5							
		s. d.					
	To 9 pair of kid gloves at	2	2	-	19	6	
	— 5 doz. pair lamb's ditto —	1	2	3	10	-	
	— 12 pieces of bobbin —	-	6	-	6	-	
					4	15	6
		25					
5	<i>Lady Strawberry, Cr.</i>						
	By cash, in full				39	-	5
		27					
5	<i>Mr. Samuel Edwards, Dr.</i>						
		d.					
	To 12 lb. of flax — at	10	-	-	10	-	
	— 14 — — —	9	-	-	10	6	
					1	-	6
		30					
5	<i>Mr. Matthew Milton, Cr.</i>						
	By 30 gal. brandy at 8s. 6d.				12	15	-
	— cash, in full				1	7	5
					14	2	5
		Nov. 4					
r	<i>Doctor Tristram Shandy, Cr.</i>						
	By cash, in full				37	15	-½
		7					
6	<i>Samuel Simpson, Esq; Dr.</i>						
	To 3 sugar loaves, wt. 32½ lb. at 8½d.				1	3	-½

Nov.

APPENDIX.

165

		Nov. 13	£.	s.	d.
1	<i>Mr. James Elford, Cr.</i>				
	By a bill for	_____	50	-	-
		_____ 15			
3	<i>Capt. James Dixon, Cr.</i>				
	By 3 pieces of holland, qt. 112½ ells, at	_____	42	3	9
	7s. 6d.	_____ 15			
3	<i>Capt. James Dixon, Dr.</i>				
	To cash, in full	_____	3	1	10½
		_____ 20			
6	<i>Samuel Simpson, Esq; Dr.</i>				
	To 15½ lb. of currants, at 4d.	_____	-	5	2
		_____ 22			
6	<i>Mr. Thomas Grey, Dr.</i>				
				s.	d.
	To 2 doz. knives and forks at	15 -	1	10	-
	— a set of china	_____	2	10	-
	— 18 china plates	_____ 2 3	2	-	6
	— 3 dishes	_____ 4 6	-	13	6
	— a mahogany tea board	_____	-	10	6
			7	4	6
		_____ 26			
6	<i>Mr. Thomas Grey, Cr.</i>				
	By 42 ells of holland at 5s. 6d.	_____	11	11	-
		_____ 28			
2	<i>Sir Jeffery Slingstone, Cr.</i>				
	By cash, in full	_____	36	1	4
		_____ 29			
6	<i>Samuel Simpson, Esq; Dr.</i>				
				s.	d.
	To 17¼ lb. malaga raisins at	- 5½	-	7	10½
	— 19¾ — raisins of the sun	- 6	-	9	10½
	— 17 — rice	- 3½	-	4	11½
	— 8½ — pepper	- 1 6	-	12	4½
	— 13 oz. cloves	- 9	-	9	9
			2	4	10½
					Dec.

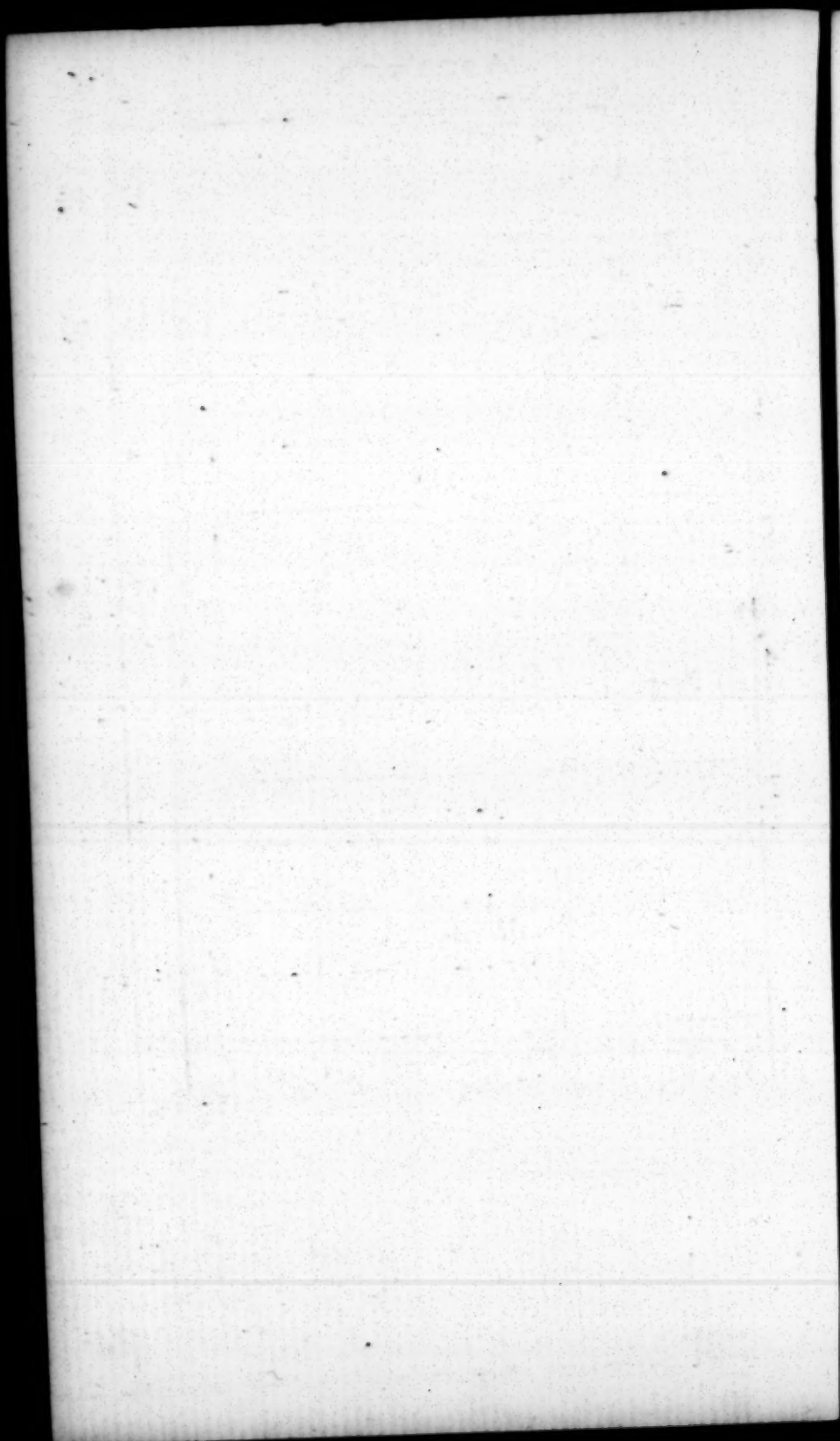
Dec. 1.		£.	s.	d.
2	<i>Mr. James Wilson, Schoolmaster, Cr.</i>			
	By cash, in full	2	8	7
3				
2	<i>Mr. Alderman Ableman, Dr.</i>			
	To a pipe of wine	25	-	-
6				
2	<i>William Winton, Esq; Cr.</i>			
	By 30 gal. brandy, at 7s. 6d.	11	5	-
	— cash, in full	9	6	0 $\frac{1}{2}$
		20	11	0 $\frac{1}{2}$
8				
6	<i>Mr. Thomas Hunter, Dr.</i>			
	To 3 $\frac{1}{2}$ chaldrons of coals at 1l. 15s.	5	5	-
10				
2	<i>Mr. William Watson, Cr.</i>			
	By cash, in full	3	17	-
12				
7	<i>Peter Thompson, of Worcester, Esq; Dr.</i>			
	To 5 butts of oil, wt. 55 cwt. 1 qr. 20 lb.			
	gross, tare 18 lb. a cwt. at 24l. 10s. a			
	tun of 236 gal. and 7 $\frac{1}{2}$ lb. neat to the			
	gallon	72	2	5 $\frac{1}{4}$
13				
6	<i>Mr Henry Forster, Cr.</i>			
	By cash, in full	2	6	10 $\frac{1}{2}$
15				
3	<i>Sir Thomas Lawson, Cr.</i>			
	By 3 c. 2 qr. 14 lb. of tobacco at 4l. a cwt.	14	10	-

Dec.

APPENDIX.

167

Dec. 18		£.	s.	d.
3	<i>Mrs. Shields, Dr.</i>			
	To a lump of sugar wt. $22\frac{1}{2}$ lb. at $8\frac{1}{2}$ d.	-	16	$1\frac{1}{2}$
20				
6	<i>Samuel Simpson, Esq; Cr.</i>			
	By cash, in full — —	3	13	$0\frac{1}{4}$
22				
4	<i>Miss Fanny Dawson, Cr.</i>			
	By cash, in full — —	4	4	"
23				
7	<i>Mr. Edward Young, Dr.</i>			
	To 3 cwt. 1 qr. cheese at 30 s. —	4	17	6
24				
4	<i>Mr. Roger Retail, Cr.</i>			
	By a bill upon Thomas Williams, Esq; for	50	-	-
3	<i>Mrs. Shields, Cr.</i>			
	By cash, in full — — —	1	1	$6\frac{1}{4}$
29				
5	<i>Mrs. March, Cr.</i>			
	By cash, in full — — —	18	-	11
30				
7	<i>Mr. Joseph King, of Windsor, Cr.</i>			
	By 18 qr. 4 bush of wheat at 1l. 8s	25	18	-



LEDGER A.

The ALPHABET.

A		B		C	
<i>Alder. Ableman</i>	2	<i>Mr. J. Baker</i>	4	<i>Mr. C. Compound</i>	4
<i>Mr. C. Anderfon</i>	4	<i>Mr. R. Barber</i>	5		
		<i>Balance</i>	7		
D		E		F	
<i>Capt. J. Dixon</i>	3	<i>Mr. J. Elford</i>	1	<i>Mr. H. Forster</i>	6
<i>Miss F. Dawson</i>	4	<i>Mr. S. Edwards</i>	5		
G		H		I	
<i>Mrs. M. Gray</i>	6	<i>Mr. T. Hunter</i>	6	<i>D. Johnson, Esq;</i>	3
<i>Mr. T. Grey</i>	6				
K		L		M	
<i>Mr. J. King</i>	7	<i>Sir T. Lawson</i>	3	<i>Sir J. Moore</i>	1
				<i>M. Masterman</i>	1
				<i>Mrs. March</i>	5
				<i>Mr. M. Milton</i>	5
N		O		P	
<i>Mr. N. Norton</i>	3				
Q		R		S	
		<i>Mr. R. Retail</i>	4	<i>D. T. Shandy</i>	1
				<i>L. Strawberry</i>	1, 5
				<i>Sir J. Slingstone</i>	2
				<i>Mrs. Shields</i>	3
				<i>S. Simpson, Esq;</i>	6
T		V		W	
<i>P. Thompson, Esq;</i>	7			<i>Mr. J. Wilson</i>	2
				<i>W. Winton, Esq;</i>	2
				<i>Mr. W. Watfen</i>	2
X		Y		Z	
		<i>Mr. E. Young</i>	7		
		Q			

Dr.

	<i>Dr.</i>	<i>Mr. James</i>	£.	s.	d.
1766					
Jan. 1	To fundries	—	32	12	6
May 3	To fundries	—	20	12	4
			—	—	—
			53	4	10
			—	—	—
1766	<i>Dr.</i>	<i>Doctor Tristram</i>			
Jan. 1	To fundries	— —	13	17	1
Mar. 22	To fundries	— —	23	17	11 $\frac{1}{2}$
			—	—	—
			37	15	0 $\frac{1}{2}$
			—	—	—
1766	<i>Dr.</i>	<i>Mrs. Mary</i>			
Jan. 4	To fundries	—	3	-	3 $\frac{1}{2}$
			—	—	—
June 17	To fundries	—	2	8	5 $\frac{1}{4}$
Sept. 18	To cash, in full	—	1	18	2 $\frac{1}{4}$
			—	—	—
			7	6	11 $\frac{1}{2}$
			—	—	—
1766	<i>Dr.</i>	<i>Lady Strawberry</i>			
Jan. 9	To fundries	—	16	2	7 $\frac{1}{2}$
Apr. 12	To fundries	—	11	17	9 $\frac{1}{2}$
Sept 4	To 12 $\frac{1}{2}$ yds fatten, at 10s. 8d.		6	10	8
12	To 11 $\frac{1}{8}$ yds. velvet, at 18s.		10	4	9
			—	—	—
			44	15	10
			—	—	—
1766	<i>Dr.</i>	<i>Sir Jonas</i>			
Jan. 20	To a ream of paper	—	1	-	-

(1)

APPENDIX.

17F

		£.	s.	d.
1766	<i>Elford, of Bath, Cr.</i>			
Nov. 12	By a bill for ———	50	-	-
	By account at fol. 1, ledger B	3	4	10
		53	4	10
<hr/>		<hr/>		
<i>Shandy, of York, Cr.</i>				
1766				
Nov. 4	By cash, in full ———	37	15	0 $\frac{1}{2}$
<hr/>		<hr/>		
<i>Masterman, Cr.</i>				
1766				
Apr. 25	By cash, in full ———	3	-	3 $\frac{1}{2}$
June 21	By 40 yds of sheeting, at 2s. 2d	4	6	8
		7	6	11 $\frac{1}{2}$
<hr/>		<hr/>		
<i>Gr.</i>				
1766				
Sept. 22	By cash, received by the steward	20	-	-
	By account at fol. 5 —	24	15	10
		44	15	10
<hr/>		<hr/>		
1766	<i>Moore, Cr.</i>			
Feb. 20	By cash, in full ———	1	-	-

172

APPENDIX

(2)

	Dr.	Mr. James	£.	s.	d.
1766					
Jan. 27	To fundries	—	-	18	5
Sept. 6	To fundries	—	1	3	7
12	To the universal penman		1	5	-
			3	7	0
<hr/>					
1766	Dr.	Mr. Alderman			
Feb. 5	To fundries	—	15	17	-
Dec. 3	To a pipe of wine	—	25	-	-
<hr/>					
1766	Dr.	William			
Feb. 12	To fundries	—	2	3	6 $\frac{1}{2}$
Aug. 1.	To fundries	—	18	7	6
			20	11	0 $\frac{1}{2}$
<hr/>					
1766	Dr.	Sir Jeffery			
Feb. 27	To fundries	—	56	1	4
<hr/>					
1766	Dr.	Mr. William			
Mar. 10	To fundries	—	3	17	-

		£.	s.	d.
1766 <i>Wilson, Schoolmaster, Cr.</i>				
July 7	By cash, in full —	-	18	5
		—	—	—
Dec. 1	By cash, in full —	2	8	7
		—	—	—
		3	7	-
		—	—	—
1766 <i>Alleman, Cr.</i>				
Aug. 7	By fundries —	15	17	6
		—	—	—
	By account at fol. 1, ledger B	25	-	-
		—	—	—
		—	—	—
<i>Winton, Esq; Cr.</i>				
		—	—	—
1766				
Dec. 6	By fundries, in full —	20	11	0 $\frac{1}{2}$
		—	—	—
1766 <i>Slingstone, Cr.</i>				
June 12	By a bank note —	20	-	-
Nov. 28	By cash, in full —	36	1	4
		—	—	—
		56	1	4
		—	—	—
1766 <i>Watson, Cr.</i>				
Dec. 10	By cash, in full —	3	17	-

174

APPENDIX.

(3)

		Dr.	Sir Thomas	£.	s.	d.
1766						
Apr. 7	To fundries	_____		27	17	-
May 10	To fundries	_____		8	12	2½
				—	—	—
				36	9	2½
<hr/>						
1766		Dr.	David			
Apr. 24	To fundries	_____		2	-	8½
May 20	To fundries	_____		17	7	10
June 28	To fundries	_____		34	12	11½
				—	—	—
				54	1	6
<hr/>						
1766		Dr.	Mr. Nicholas			
May 14	To fundries	_____		23	15	4½
<hr/>						
1766		Dr.	Mrs.			
May 26	To fundries	_____		-	5	5½
Dec. 18	To 22¼ lb. sugar, at 8½ d.			"	16	1½
				—	—	—
				1	1	6½
<hr/>						
1766		Dr.	Capt. James			
June 3	To fundries	_____		30	5	5½
July 28	To fundries	_____		8	16	5
Nov. 15	To cash, in full	_____		3	1	10½
				—	—	—
				42	3	9

	<i>Lawson, Cr.</i>	£.	s.	d.
1766				
June 3	By a bill on Capt. James Dixon	10	-	-
Dec. 15	By 3c. 2qr. 14lb. tobacco, at 4l.	14	10	-
	By account at fol. 1, ledg. B	11	19	2½
		36	9	2½
<hr/>				
1766	<i>Johnson, Esq; Cr.</i>			
Aug. 16	By cash, in part	50	-	-
Oct. 6	By cash, in full	4	1	6
		54	1	6
<hr/>				
1766	<i>Norton, of Durham, Cr.</i>			
Sept. 9	By a bank note	20	-	-
	By account at fol. 1, ledg. B*	3	15	4½
		23	15	4½
<hr/>				
	<i>Shields, Cr.</i>			
1766				
Dec. 24	By cash, in full	1	1	6¼
<hr/>				
	<i>Dixon, Cr.</i>			
1766				
Nov. 15	By 112½ ells of holland at 7s. 6d.	42	3	9

Dr.

176

APPENDIX.

(4)

			£.	s.	d.
1766	<i>Dr.</i>	<i>Miss Fanny</i>			
July 3	To fundries	_____	4	4	-
<hr/>					
1766	<i>Dr.</i>	<i>Mr. Roger Retail,</i>			
July 10	To fundries	_____	83	5	9
<hr/>					
1766	<i>Dr.</i>	<i>Mr. Charles</i>			
July 17	To fundries	_____	11	13	-
24	To fundries	_____	12	4	2
Aug. 12	To fundries	_____	5	10	6
			29	7	8
<hr/>					
1766	<i>Dr.</i>	<i>Mr. Conrade</i>			
Aug. 18	To fundries	_____	56	19	5
<hr/>					
1766	<i>Dr.</i>	<i>Mr. John</i>			
Aug. 26	To fundries	_____	16	7	-

		f.	s.	d.
1766	<i>Dawson, of Liverpool, Cr.</i>			
Dec. 22	By cash, in full ———	4	4	-
<hr/>				
1766	<i>of Newcastle upon Tyne, Cr.</i>			
Dec. 24	By a bill ———	50	-	-
	By account at fol. 1. ledg. B	33	5	9
		83	5	9
<hr/>				
1766	<i>Anderson, Cr.</i>			
Aug. 21	By 5 pockets of hops at 28s.	12	-	-
Sept. 24	By cash, in full ———	17	7	8
		29	7	8
<hr/>				
	<i>Compound, of Exeter, Cr.</i>			
	By account at fol. 2, ledg. B	56	19	5
<hr/>				
1766	<i>Baker, Cr.</i>			
Oct. 23	By cash, in part ———	10	-	-
	By account at fol. 2, ledg. B	6	7	-
		16	7	0

Dr

		£.	s.	d.
1766	<i>Dr. Mrs. March,</i>			
Sept. 3	To 8 farcenet hoods at 4s. 3d.	1	14	-
17	To 17 india fans at 3s. 10d.	3	5	2
29	To fundries	8	6	3
Oct. 23	To fundries	4	15	6
		18	-	11
<hr/>				
1766	<i>Dr. Mr. Samuel</i>			
Oct. 2	To 14 lb. flax, at 1s.	-	14	-
27	To fundries	1	-	6
		1	14	6
<hr/>				
	<i>Dr. Mr. R. Barber,</i>			
	To account at fol. 2, ledg. B	18	15	-
<hr/>				
1766	<i>Dr. Lady</i>			
	To account from fol. 1	24	15	10
Oct. 6	To 27½ yds. holland, at 5s. 6d.	7	11	3
13	To 40 yds. irish cloth, at 3s. 4d.	6	13	4
		39	-	5
<hr/>				
1766	<i>Dr. Mr. Matt. Milton,</i>			
Oct. 10	To fundries	14	2	5

		<i>of Chester, Cr.</i>	£.	s.	d.
1766	Dec 6	By cash, in full —	18	-	11
		<i>Edwards, Cr.</i>			
		By account at fol. 2, ledg. B	1	14	6
1766	Oct. 4	<i>of Bristol, Cr.</i> By 40 reams of foolscap, at 12s. 6d. — —	18	15	-
		<i>Strawberry, Cr.</i>			
1766	Oct. 25	By cash, in full —	39	-	5
1766	Oct. 30	<i>of Norwich, Cr.</i> By sundries —	12	2	5
					Dr.

180

APPENDIX.

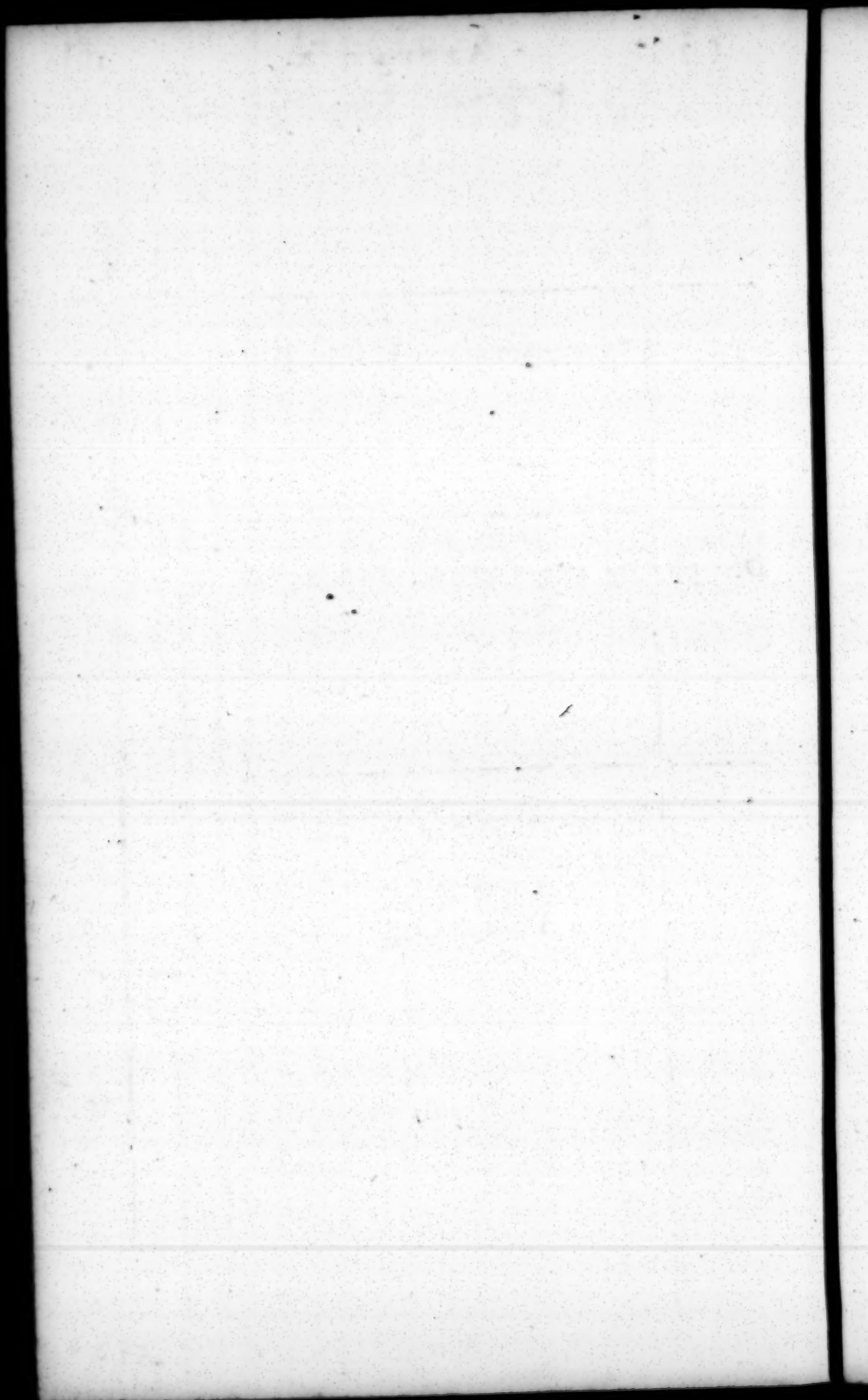
(6)

		£.	s.	d.
1766	<i>Dr. Mr. Henry</i>			
Oct. 15	To $2\frac{1}{2}$ cwt. iron, at 18s. 9d.	2	6	10 $\frac{1}{2}$
<hr/>				
	<i>Dr. Mrs. Mary</i>			
	To account at fol. 2, ledg. B.	9	8	6
<hr/>				
1766	<i>Dr. Samuel</i>			
Nov. 7	To $32\frac{1}{2}$ lb. sugar, at 8 $\frac{1}{2}$ d.	1	3	0 $\frac{1}{2}$
20	To $15\frac{1}{2}$ lb. currants, at 4 d.	-	5	2
29	To fundries	2	4	10 $\frac{1}{2}$
		3	13	0 $\frac{1}{2}$
<hr/>				
1766	<i>Dr. Mr. Thomas</i>			
Nov. 22	To fundries	7	4	6
	To account at fol. 3, ledg. B	4	6	6
		11	11	-
<hr/>				
1766	<i>Dr. Mr. Thomas</i>			
Dec. 8	To 3 chald. coals, at 1l. 15s.	5	5	-

1766	<i>Forster, Cr.</i>	£.	s.	d.
Dec. 13	By cash, in full —	2	6	10 $\frac{1}{2}$
1766	<i>Gray, Cr.</i>			
Oct. 21	By 3 ps. irish cloth, qt. 87 yds. at 2s. 2d. — —	9	8	6
1766	<i>Simpson, Esq; Cr.</i>			
Dec. 20	By cash, in full —	3	13	0 $\frac{1}{2}$
1766	<i>Grey, Cr.</i>			
Nov. 26	By 42 ells of holland, at 5s. 6d.	11	11	
	<i>Hunter, Cr.</i>			
	By account at fol. 3, ledger B	5	5	

		£.	s.	d.
1766 Dec. 12	<i>Dr. Peter Thompson,</i> To 5 butts of oil	72	2	5½
1766 Dec. 23	<i>Dr. Mr. Edward</i> To 3 c. 1 qr. cheefe, at 30s.	4	17	6
	<i>Dr. Mr. Joseph King,</i> To account at fol. 3, ledg. B	25	18	-
	<i>Dr. Balance</i> To Mr. J. Elford, due to me,	3	4	10
	To Mr. Alderman Ableman	25	-	-
	To Sir Thomas Lawfon	11	19	2½
	To Mr. Nicholas Norton	3	15	4½
	To Mr. Roger Retail	33	5	9
	To Mr. Conrade Compound	56	19	5
	To Mr. John Baker	6	7	-
	To Mr. Samuel Edwards	1	14	6
	To Mr. Thomas Hunter	5	5	-
	To Peter Thompson, Esq;	72	2	5½
	To Mr. Edward Young	4	17	6
		224	11	0¼

		£.	s.	d.
<i>of Worcester, Esq; Cr. -</i>				
	By account at fol. 3, ledg. B	72	2	5 $\frac{1}{4}$
<hr/>		<hr/>	<hr/>	<hr/>
<i>Young, Cr.</i>				
	By account at fol. 3, ledg. B	4	17	6
<hr/>		<hr/>	<hr/>	<hr/>
1766	<i>of Windsor, Cr.</i>			
Dec. 30	By 18 qr. 4 bush. of wheat, at 1 <i>l.</i> 8 <i>s.</i>	25	18	-
<hr/>		<hr/>	<hr/>	<hr/>
<i>Cr.</i>				
	By Mr. R. Barber —	18	15	-
	By Mrs. Mary Gray —	9	8	6
	By Mr. Joseph King —	25	18	-
	By Mr Thomas Grey —	4	6	6
		<hr/>	<hr/>	<hr/>
		58	8	-



LEDGER B.

The ALPHABET.

A	B	C
<i>Alder. Ableman</i> 1	<i>Mr. J. Baker</i> 2 <i>Mr. R. Barber</i> 2	<i>Mr. C. Compound</i> 2
D	E	F
	<i>Mr. J. Elford</i> 1 <i>Mr. S. Edwards</i> 2	
G	H	I
<i>Mrs. M. Gray</i> 2 <i>Mr. T. Grey</i> 3	<i>Mr. T. Hunter</i> 3	
K	L	M
<i>Mr. J. King</i> 3	<i>Sir T. Lawson</i> 1	
N	O	P
<i>Mr. N. Norton</i> 1		
Q	R	S
	<i>Mr. R. Retail</i> 1	
T	V	W
<i>P. Thompson, Esq;</i> 3		
X	Y	Z
	<i>Mr. E. Young</i> 3	
	R 3	Dr.

	<i>Dr.</i>	<i>Mr. James</i>	£.	s.	d.
1767	To account at fol. 1, ledg. A		3	4	10
<hr/>					
1767	<i>Dr.</i>	<i>Mr. Alderman</i>			
	To account at fol. 2, ledg. A		25	-	-
<hr/>					
1767	<i>Dr.</i>	<i>Sir Thomas</i>			
	To account at fol. 3, ledg. A		11	19	2½
<hr/>					
1767	<i>Dr.</i>	<i>Mr. Nicholas</i>			
	To account at fol. 3, ledg. A		3	15	4½
<hr/>					
1767	<i>Dr.</i>	<i>Mr. Roger Retail,</i>			
	To account at fol. 4, ledg. A		33	5	9

(1)

APPENDIX.

187

Elford, of Bath, Cr.

£. s. d.

Ableman, Cr.

Lawson, Cr.

Norton, of Durham, Cr.

of Newcastle upon Tyne, Cr.

Dr.

	<i>Dr.</i>	<i>Mr. Conrade</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>
1767	To account at fol. 4, ledg. A		56	19	5
<hr/>					
	<i>Dr.</i>	<i>Mr. John</i>			
1767	To account at fol. 4, ledg. A		6	7	-
<hr/>					
	<i>Dr.</i>	<i>Samuel</i>			
1767	To account at fol. 5, ledg. A		1	14	6
<hr/>					
	<i>Dr.</i>	<i>Mr. R. Barber,</i>			
<hr/>					
	<i>Dr.</i>	<i>Mrs. Mary</i>			
<hr/>					

(2)

APPENDIX.

189

Compound, of Exeter, Cr.

£. s. d.

*Baker, Cr.**Edwards, Cr.**of Bristol, Cr.*

1767

By account at fol. 5, ledg. A

18 15 -

Gray, Cr.

1767

By account at fol. 6, ledg. A

9 8 6

Dr.

	<i>Dr.</i>	<i>Mr. Thomas</i>	<i>£.</i>	<i>s.</i>	<i>d.</i>
1767	<i>Dr.</i>	<i>Mr. Thomas</i> To account at fol. 6, ledg. A	5	5	-
1767	<i>Dr.</i>	<i>Peter Thompson,</i> To account at fol 7, ledg. A	72	2	5 $\frac{1}{4}$
1767	<i>Dr.</i>	<i>Mr. Edward</i> To account at fol. 7, ledg. A	4	17	6
	<i>Dr.</i>	<i>Mr. Joseph King,</i>			

(3)

APPENDIX:

191 .

	<i>Grey, Cr.</i>	£.	s.	d.
1767	By account at fol. 6, ledg. A	4	6	6
	<i>Hunter, Cr.</i>			
	<i>of Worcester, Esq; Cr.</i>			
	<i>Young, Cr.</i>			
1767	<i>of Windsor, Cr.</i> By account at fol. 7, ledg. A	25	18	-
THE END.				

ERRATA.

The *Ans.* to quest. 4, pag. 54, must be $4\frac{93}{17}$.

Ans. to quest. 6, pag. 144, ought to be 39027●.

THE EXPERIENCED
English Housekeeper,

FOR THE USE AND EASE OF
Ladies, Housekeepers, Cooks, &c.

Written purely from PRACTICE,

DEDICATED TO THE

Hon. Lady ELIZABETH WARBURTON,

Whom the Author lately served as Housekeeper :

Consisting of several Hundred Original Receipts, most of which
never appeared in Print.

PART I. Lemon Pickle, Browning
for all Sorts of Made Dishes, Soups,
Fish, Plain Meat, Game, Made
Dishes both hot and cold, Pies,
Puddings, &c.

PART II. All Kinds of Confectionary,
particularly the Gold and Silver Web
for covering of Sweetmeats, and a Desert
of Spun Sugar, with Directions to set out
a Table in the most elegant Manner and

in the modern Taste, Floating
Islands, Fish-Ponds, Transparent
Puddings, Trifles, Whips, &c.

PART III. Pickling, Potting, and
Collaring, Wines, Vinegars, Catch-
ups, Distilling, with two most valuable
Receipts, one for refining Malt-Liquors,
the other for curing Acid Wines, and a
correct List of every Thing in Season
for every Month in the Year.

By ELIZABETH RAFFALD.

A NEW EDITION,

In which are inserted some celebrated Receipts by other Modern Authors.

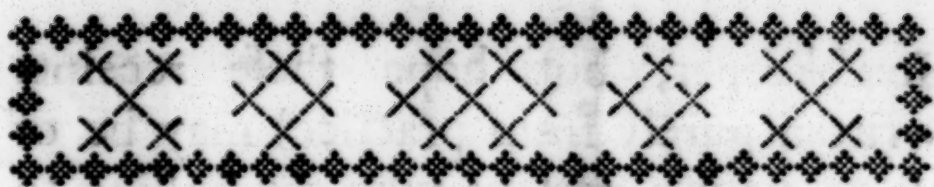
L O N D O N :

PRINTED FOR THE BOOKSELLERS,

MDCCLXXXIX.

Johnson & 1293





TO THE HONOURABLE

LADY ELIZABETH WARBURTON.

PERMIT me, honoured Madam, to lay before you a work, for which I am ambitious of obtaining your Ladyship's approbation, as much as to oblige a great number of my friends, who are well acquainted with the practice I have had in the Art of Cookery, ever since I left your Ladyship's family, and have often solicited me to publish for the instruction of their housekeepers.

As I flatter myself I had the happiness of giving satisfaction, during my service, Madam, in your family, it would be a still greater encouragement, should my endeavours, for the service of the sex, be honoured with the favourable opinion of so good a judge of propriety and elegance as your Ladyship.

I am not vain enough to propose adding any thing to the Experienced

DEDICATION.

Housekeeper, but hope these receipts (written purely from practice) may be of use to young persons who are willing to improve themselves.

I rely on your Ladyship's candour, and whatever Ladies favour this book with reading it, to excuse the plainness of the stile; as, in compliance with the desire of my friends, I have studied to express myself so as to be understood by the meanest capacity, and think myself happy in being allowed the honour of subscribing,

M A D A M,

Your Ladyship's

Most dutiful,

Most obedient,

And most humble Servant,

ELIZABETH RAFFALD.

Preface to the First Edition.

WHEN I reflect upon the number of books already in print upon this subject, and with what contempt they are read, I cannot but be apprehensive that this may meet the same fate with some, who will censure before they either see it or try its value.

Therefore the only favour I have to beg of the public is, not to censure my work before they have made trial of some one receipt, which I am persuaded, if carefully followed, will answer their expectations; as I can faithfully assure my friends, that they are truly written from my own experience, and not borrowed from any other author, nor glossed over with hard names, or words of high stile, but written in my own plain language, and every sheet carefully perused as it came from the press, having an opportunity of having it printed by a neighbour, whom I can rely on doing it the strictest justice, without the least alteration.

The whole work being now completed to my wishes, I think it my duty to render my most sincere and grateful thanks to my most noble and worthy friends, who have already shewn their good opinion of my endeavours to serve my sex, by raising me so large a subscription, which far exceeds my expectations. I have not only been honoured by having above eight hundred of their names inserted in my subscription, but also have had all their interest in this laborious undertaking, which I have at last arrived to the happiness of completing, though at
the

the expence of my health, by being too studious, and giving too close application.

The only anxious wish I have left is, that my worthy friends may find it useful in their families, and be an instructor to the young and ignorant, as it has been my chiefest care to write in as plain a stile as possible, so as to be understood by the weakest capacity.

I am not afraid of being called extravagant, if my reader does not think that I have erred on the frugal hand.

I have made it my study to please both the eye and the palate, without using pernicious things for the sake of beauty.

And though I have given some of my dishes French names, as they are only known by those names, yet they will not be found very expensive, nor add compositious but as plain as the nature of the dish will admit of.

The receipts for the confectionary are such as I daily see in my own shop, which any Lady may examine at pleasure, as I still continue my best endeavours to give satisfaction to all who are pleased to favour me with their custom.

It may be necessary to inform my readers, that I have spent fifteen years in great and worthy families, in the capacity of a House-keeper, and had an opportunity of travelling with them; but finding the common servants generally so ignorant in dressing meat, and a good cook so hard to be met with, put me upon studying the Art of Cookery more than perhaps
I other-

I otherwise should have done ; always endeavouring to join œconomy with neatness and elegance, being sensible what valuable qualifications these are in a housekeeper or cook ; for of what use is their skill, if they put their master or lady to an immoderate expence in dressing a dinner for a small company, when at the same time a prudent manager would have dressed twice the number of dishes for a much greater company, at half the cost ?

I have given no directions of cullis, as I have found, by experience, that lemon pickle and browning answers both for beauty and taste (at a trifling expence) better than cullis, which is extravagant ; for had I known the use and value of those two receipts when I first took upon me the part and duty of a housekeeper, it would have saved me a great deal of trouble in making gravy, and those I served a deal of expence.

The number of receipts in this book are not so numerous as in some others, but they are what will be found useful and sufficient for any Gentleman's family——neither have I meddled with physical receipts, leaving them to the physicians' superior judgement, whose proper province they are.

Description

Description of the P L A T E.

THE Plate is the design of three stove-fires for the kitchen, that will burn coals or embers instead of charcoal, (which I always found expensive as well as pernicious to the cooks) and will carry off the smoke of the coals and steam, and smell of the pots and stew-pans; the coals are burnt in cast iron pots, flat at the bottom, with bars.

AA, Fronts of the stove.

BB, Top of the stove, which is covered all over with cast iron.

CC, Stove-pots, in which the fire is made.

D, The form of the pot, with two vents cast in them, six inches deep at the top, and three wide, as expressed at HH in the pot, and to let the smoke through at H's in the flues.

EE, Carried from the fire through the back wall to the kitchen chimney, as expressed in the lower plan.

FF, Back wall.

G, The chimney-breast, betwixt which and the back wall the steam rises, and goes off into the kitchen chimney by a vent made into it.

HH, Vents in the pot.

II, Draughts for the fires, and to receive the ashes.

The scale will give the dimensions.